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The Environment and Directed Technical Change

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Abstract

This paper introduces endogenous and directed technical change in a growth model with environmental constraints and limited resources. A unique final good is produced by combining inputs from two sectors. One of these sectors uses “dirty” machines and thus creates environmental degradation. Research can be directed to improving the technology of machines in either sector. We characterize dynamic tax policies that achieve sustainable growth or maximize intertemporal welfare, as a function of structural characteristics of the economy, in particular the degree of substitutability between clean and dirty inputs, environmental and resource stocks, and cross-country technological spillovers. First, we find that factoring in directed technical change: (i) increases the cost of delaying intervention, particularly when the inputs are highly substitutable; (ii) calls for the use of profit taxes or other instruments to direct innovation, in addition to the input (carbon) tax emphasized in the literature. Second, we show that: (i) in the case where the inputs are sufficiently substitutable, sustainable long-run growth can be achieved with temporary taxation of dirty innovation and production; (ii) the sooner and the stronger is the policy response, the shorter is the slow growth transition phase; (iii) the use of an exhaustible resource in dirty input production helps the switch to clean innovation under laissez-faire when the two inputs are substitutes. Third, we find that under reasonable parameter values (corresponding to those used in existing models with exogenous technology) and with sufficient substitutability between inputs, it is optimal to redirect technical change towards clean technologies immediately. Finally, in a two-country extension of the baseline model where: (a) the two inputs are sufficiently substitutable in both countries; (b) dirty input production in both countries depletes the global environmental stock; (c) the South imitates technologies invented in the North, we show that taxing dirty innovation and production in the North only may be sufficient to avoid a global disaster, but international trade increases the need for global policy coordination.

JEL Classification: O30, O31, O33, C65.

Keywords: environment, exhaustible resources, directed technological change, innovation.

Work in Progress. Comments Welcome.

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1 Introduction

How to control and limit climate change caused by our growing consumption of fossil fuels and to develop alternative energy sources to these fossil fuels are among the most pressing policy challenges facing the world today. While climate scientists have focused on various aspects of the damage that our current energy consumption causes to the environment,¹ economists have emphasized both the benefits—in terms of limiting environmental degradation—and costs—in terms of reducing economic growth—of different policy proposals. More importantly, while a large part of the discussion among climate scientists focuses on the effect of various policies on the development of alternative—and more “environmentally friendly”—energy sources, the response of technological change to environmental policy has until very recently been all but ignored by leading economic analyses of environment policy, which have mostly focused on computable general equilibrium models with exogenous technology.² This omission is despite the fact that existing empirical evidence indicates that changes in the relative price of energy inputs have an important effect on the types of technologies that are developed and adopted. For example, Newell, Jaffe and Stavins (1999) show that between 1960 and 1980, when energy prices were stable, innovations in air-conditioning have reduced the prices faced by consumers following the oil price hikes, these air conditioners became more energy efficient. Popp (2002) provides more systematic evidence on the same point by using patent data from 1970 to 1994; he documents the impact of energy prices on patents for energy-saving innovations.³

This paper is motivated by our belief that a satisfactory framework for the study of the costs and benefits of different environmental policies must include, at its centerpiece, the endogenous response of different types of technologies to proposed policies. Our purpose is to take a first step towards the development of such a framework. We propose a simple two-sector model of directed technical change. The unique final good is produced by combining the inputs produced by these two sectors. One of them uses “dirty” machines and creates environmental degradation. Profit-maximizing researchers build on previous innovations (“build on the shoulders of giants”) and direct their research to improving the quality of machines in one or the other sector. We first focus on a single (and closed) economy.

Our framework highlights the central roles played by the *market size* and the *price effects* on the direction of technical change (Acemoglu, 1998, 2002).⁴ The market size effect encourages innovation towards the larger input sector, while the price effect makes innovation directed

¹See, for instance, Stott et al. (2004) on the contribution of human activity to the European heatwave of 2003, Emanuel (2005) and Landsea (2005) on the increased impact and destructiveness of tropical cyclones and Atlantic hurricanes over the last decades, and Nicholls and Lowe (2006) on sea-level rise.

²See, e.g., Nordhaus (1994), MacCracken et al. (1999), Nordhaus and Boyer (2000).

³Acemoglu and Finkelstein (2008) provide evidence from the health-care sector, suggesting that capital investments and technology adoption are highly responsive to changes in relative prices caused by regulation.

⁴In addition, we will see that what is referred to as “state dependence” in Acemoglu (2002), where current research builds on past research in the same sector, plays an important role here.

towards the sector with higher price. The relative magnitudes of these effects in our framework are, in turn, determined by three factors:

1. The elasticity of substitution between the two sectors.
2. The relative levels of development of the technologies of the two sectors.
3. Whether dirty inputs are produced using an exhaustible resource.

Because of the environmental externality, the decentralized equilibrium is not optimal. Moreover, the laissez-faire equilibrium typically leads to an “environmental disaster,” where the quality of the environment falls below a critical threshold (the most important exception to this case is when the dirty input uses an exhaustible resource, in which case profit incentives in the laissez-faire equilibrium may be sufficient to prevent such an environmental disaster).

More interesting are the results on what types of policies can prevent such disasters and how costly delaying their introduction is. These also depend on the market size and price effects, and thus on the three factors highlighted above. When the two sectors (clean and dirty inputs) are highly substitutable (“strong substitutes”), the economy rapidly heads towards an environmental disaster. Broadly speaking, in this case directed technical change, in particular the market size effect, implies that an initial productivity advantage in dirty inputs will induce researchers to target innovation to these machines (as the expected profit from innovating in a sector is increasing in employment and productivity in this sector) despite the price effect (i.e., the fact that the use of the more productive dirty machines also results in a lower relative price of the dirty input). However, a *temporary* policy intervention (for example, a temporary tax on the use of dirty inputs, reminiscent to a carbon tax, or simply a profit tax in dirty sectors/subsidy on clean innovation) may be sufficient prevent an environmental disaster when the two sectors are highly substitutable. Such a temporary intervention would *redirect* technical change, meaning that it would induce research (and production) to shift away from the dirty input. Once the clean inputs become developed, the tax is no longer required. But this case also highlights that delaying the introduction of such policies is potentially quite costly; delay would increase the gap between clean and dirty sectors and thus call for higher taxes (and for more extended slowdown in the economy) in the future in order to avoid a disaster.⁵

In contrast, when the two sectors are not sufficiently substitutable, an environmental disaster develops less rapidly (and depending on the specification of environmental dynamics may also be less likely) in the laissez-faire equilibrium, since in that case the price effect dominates

⁵The costs of delay and the need for rapid and decisive action in our framework (with sufficient substitutability) contrast sharply with existing policy recommendations from computable general equilibrium models with exogenous technology discussed below, which suggest that policy should be gradual. We return to this issue in greater detail in subsection 2.4.3.

and the initial productivity advantage in dirty machines induces innovation in the more backward, clean sector. But preventing such a disaster now requires a permanent policy intervention (because it is now impossible to switch research permanently to the clean sector).

In addition to showing how simple policy interventions can prevent environmental disasters, we also use this framework to characterize the structure of optimal environmental regulation. We show that optimal regulation involves the use of both input taxes (such as the carbon tax) and profit taxes on the dirty sector (or equivalently, research subsidies to clean research). Even though input taxes both discourage polluting activities and make research in the clean sector more profitable, sole reliance on input (carbon) taxes would cause excessive production distortions in the short run. The use of profit taxes lessens this reliance on, and the short-run negative effects of, input taxes. We also show that when dirty and clean inputs are strong substitutes, optimal environmental regulation involves profit and input taxes only for a temporary period, similar to the temporary use of profit taxes for avoiding environmental disasters. This is again thanks to the fact that when clean inputs become sufficiently advanced, both research and production switch to that sector. In contrast, with lesser substitutability between the two sectors, optimal environmental regulation involves permanent input and profit taxes.

As a first step towards a more careful quantitative analysis of environmental policy in the presence of directed technical change, we also perform a simple calibration exercise. We relate our environmental quality variable to temperature and atmospheric concentration of carbon. We choose a parametrization such that the costs of small increases in temperature are comparable to those obtained in Nordhaus's DICE 2007 model. Given such a parametrization, we find that, in the presence of directed technical change, for high (but reasonable) elasticities of substitution between clean and dirty inputs (nonfossil and fossil fuels), the optimal policy involves an immediate switch of all R&D effort to clean technologies (but with our baseline elasticity, it takes about seven decades for 90% of production to switch to clean technologies). Interestingly, the general quantitative structure of optimal environmental policy appears broadly robust to whether one uses a low or medium discount rate (which is the main source of the different conclusions on optimal environmental policy in the Stern report or in Nordhaus's research).

The degree of substitution, which plays a central role in the model, has a clear empirical counterpart. For example, renewable energy, provided it can be stored and transported efficiently would be highly substitutable with energy derived from fossil fuels. This reasoning would suggest a (very) high degree of substitution between dirty and clean inputs, since the same production services can be obtained from alternative energy with less pollution. In contrast, if the "clean alternative" were to reduce our consumption of energy permanently, for example by using less effective transport technologies, this would correspond to a low degree of substitution, since greater consumption of non-energy commodities would increase the de-

mand for energy. Moreover, this parameter, though not systematically investigated by existing research, can be estimated in future empirical work and should become a crucial input into the design of environmental policy.

Our framework also clearly illustrates the effects of exhaustibility of resources on the *laissez-faire* equilibrium and on the structure of optimal policy. An environmental disaster is less likely when the dirty sector uses an exhaustible resource (and the two sectors have a high degree of substitution), since this will create a tendency for its costs to increase as this resource is depleted. The greater cost of the resource will induce an increase in the price and a relative contraction in the size of the dirty sector. With a high elasticity between the two inputs, the market size effect then encourages innovation directed towards the clean sector, even without government intervention, potentially avoiding an environmental disaster. The contrast between exhaustible and non-exhaustible resources in the model also has a clear empirical counterpart.

Finally, our framework can be used to analyze issues of global environmental policy coordination in a multi-country setting. Key questions in this case include: (i) whether environmental degradation (and in the extreme case, a disaster) can be avoided by policies in the “North” alone, that is, without global policy coordination imposing similar environmental regulations in the South, (i.e., in developing countries such as India and China); (ii) what the effects of international trade are on the development of environmental technologies and on sustainability. Our framework provides new answers to these questions. Again, the three factors emphasized above turn out to be central. When the two sectors are highly substitutable (and there are international technology linkages), environmental regulation only in the North can be sufficient to stave off an environmental disaster, because once these policies induce a sufficient improvement in the technology of the clean sector, the South will also adjust its technology and pattern of production. However, international trade, without global policy coordination, may lead to increased environmental damage. In particular, it creates a “pollution haven” in the South, allowing firms there to specialize in, and increase the production of, the dirty inputs, which is not taxed there. This increases the need for global policy coordination for avoiding an environmental disaster.

Our paper relates to the substantial and growing literature on growth, resources, and the environment. Nordhaus’s (1994) pioneering study proposed a dynamic integrated model of climate change and the economy (the DICE model), which extends the neoclassical Ramsey model with equations representing geophysical relationships (emissions equations, concentrations equations, climate-change equations, climate-damage equations), and their interactions with economic outcomes.⁶ Recent work by Golosov, Hassler, Krusell and Tsyvinski (2009)

⁶Nordhaus and Boyer (2000) extend the DICE model to include eight regions making decisions independently (the “RICE” model, or Regional Dynamic Integrated model of Climate and the Economy). The analysis of economic activity and its consequences in terms of climate change using this type of approach has been the subject of an extensive report conducted by Stern (2006).

characterizes the structure of optimal policies in a related model with exogenous technology and exhaustible resources. In our calibration exercise we build on Nordhaus’s study and results. There is also a growing literature focusing on climate’s impact on different types of outcomes, such as health, agriculture, conflict, and economic growth.⁷ Another branch of the literature focuses on the measurement of the costs of climate change, particularly stressing issues related to risk, uncertainty and discounting.⁸ Based on the assessment of discounting and related issues, this literature has prescribed either decisive and immediate governmental action (for example, Stern, 2006) or a more gradualist approach, with modest control in the short-run followed by sharper emissions reduction in the medium and the long run.⁹ Finally, some authors have built on Weitzman (1974)’s analysis on the use of price or quantity instruments to study climate change policy and the choice between taxes and quotas.¹⁰

The response of technology to environmental degradation and environmental policy, our main focus in this paper, has received much less attention in the economics literature, however. Early work by Stokey (1998) highlighted the tension between growth and the environment, and showed that degradation of the environment can create an endogenous limit to growth. Aghion and Howitt (1998, Chapter 5) introduced environmental constraints in a Schumpeterian growth model and emphasized that environmental constraints may not prevent sustainable long-run growth when environment-saving innovations are allowed. Neither of these early contributions allowed technological change to be directed to clean or dirty technologies.¹¹

Subsequent work by Popp (2004) allowed for directed innovation in the energy sector.¹² Popp presents a calibration exercise and establishes that models that ignore the directed technical change effects can significantly overstate the cost of environmental regulation. While Popp’s work is highly complementary to ours, neither his work nor others develop a systematic framework for the analysis of the impact of environmental regulations on the direction of

⁷See, for example, Deschenes and Moretti (2007), Sachs and Malaney (2002), Deschenes and Greenstone (2007), Miguel et al. (2004), and Dell et al. (2008).

⁸For example, Stern (2006), Weitzman (2007, 2008), Dasgupta (2007, 2008), Nordhaus (2007), von Below and Persson (2008), Mendelsohn (2007), and Tol and Yohe (2006).

⁹See, for example, the work by Nordhaus and coauthors (1994, 2000, 2002). A survey of the results of greenhouse-gas stabilization policy in several climate-change models can be found in Energy Modeling Forum Study 19 (2004).

¹⁰See for example Hepburn (2006) and Pizer (2002). In addition, several studies address the importance of internationally coordinated policy, such as Stern (2006) and Watson (2001). Aldy et al. (2003) provide a comparison of the different architectures for global climate policy.

¹¹First attempts at introducing endogenous directed technical change in models of growth and the environment also build on Acemoglu (1998, 2002) and include Grubler and Messner (1998), Manne and Richels (2002), Messner (1997), Buonanno et al (2003), Nordhaus (2002), Sue Wing (2003), and Di Maria and Valente (2006), and more closely related to the model here, Aghion and Howitt (2009, Chapter 16).

¹²Nordhaus (2002) also extends the R&DICE model by including a simple form of induced technical change. In particular, he uses a variant of his previous framework with fixed proportions, in which R&D is modeled as shifting the minimum level of carbon/energy inputs required for production. However, since factor substitution is not allowed in the model, it is not possible to compare the role of induced innovation with that of factor substitution in reducing greenhouse emissions. Popp’s (2004) ENTICE model allows for both endogenous technological change and factor substitution.

technological change. We develop a general and tractable framework, extending the models in Acemoglu (1998, 2002), that allows us: (i) to perform systematic comparative analyses for the effects of different types of policies on innovation, growth and environmental resources both with and without directed technical change; (ii) to study the implications of dirty inputs using exhaustible resources; (iii) to characterize dynamic optimal policy; and (iv) to study the role of international linkages in technology and trade on the effects of environmental regulations.

More recently, Gans (2009) develops a two-period model based on Acemoglu (2009b) to discuss the Porter hypothesis, that environmental regulation can lead to faster technological progress (see also Rauscher, 2009). In particular he shows that this would require a high degree of substitutability between clean and dirty inputs. We abstract from this channel in the current paper by assuming that the total R&D resources in the economy are constant, focusing instead on long-run growth sustainability and the characterization of dynamic optimal policies.

The remainder of the paper is organized as follows. Section 2 introduces our basic framework without exhaustible resources and presents the majority of our main results. In particular, it characterizes the laissez-faire equilibrium and shows how this can lead to an environmental disaster. It then shows how simple policy interventions can prevent environmental disasters and clarifies the role of directed technical change in these results. Finally, this section characterizes the structure of optimal environmental policy in this setup and also provides a preliminary quantitative assessment of how directed technical change affects the structure of optimal policy under reasonable parameter values. Section 3 introduces exhaustible resources and shows how this changes the conclusions from Section 2. Section 4 discusses issues of global policy using a two-country case extension. Section 5 concludes, while the appendices contain the proofs of several results stated in the text.

2 Baseline Model: Non-Exhaustible Resource

In this section, we introduce the baseline framework (without an exhaustible resource). We identify the market size and price effects on the direction of technical change and characterize the equilibrium of the economy under laissez-faire. We then discuss how policy interventions may be necessary to avoid “environmental disasters”, and the costs of delayed intervention. Finally, we fully characterize the optimal policy in this environment and provide a first attempt at a simple calibration to understand how the presence of directed technical change affects the structure of optimal policy under reasonable parameter values.

2.1 Preferences, Production and the Environment

We consider an infinite horizon discrete time economy inhabited by a continuum of households of mass 1 (making up the workers, entrepreneurs and scientists). We assume that all households

have preferences (or that the economy admits a representative household with preferences):

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \quad (1)$$

where C_t is consumption of the unique final good at time t , S_t denotes the *quality of the environment* at time t , and $\rho > 0$ is the discount rate. We assume that $S_t \in [0, \bar{S}]$, where \bar{S} is the quality of the environment absent any human pollution, and to simplify the notation, we also assume that it is the initial level of quality, that is, $S_0 = \bar{S}$.

The instantaneous utility function $u(C, S)$ is increasing both in C and S , twice differentiable and jointly concave in (C, S) . Moreover, we impose the following Inada-type conditions:

$$\lim_{C \downarrow 0} \frac{\partial u(C, S)}{\partial C} = \infty, \lim_{S \downarrow 0} \frac{\partial u(C, S)}{\partial S} = \infty, \text{ and } \lim_{S \downarrow 0} u(C, S) = -\infty. \quad (2)$$

The last two conditions imply that the quality of the environment reaching its lower bound has severe utility consequences.¹³ Finally we assume that

$$\lim_{S \uparrow \bar{S}} \frac{\partial u(C, \bar{S})}{\partial S} \equiv \frac{\partial u(C, \bar{S})}{\partial S} = 0, \quad (3)$$

which implies that as S approaches \bar{S} , the value of the marginal increase in environmental quality is small. This assumption is adopted to simplify the characterization of optimal environmental policy in subsection 2.4, and we discuss below how relaxing it affects the results.

There is a unique final good, produced competitively from the output of two intermediate sectors, according to the aggregate production function

$$Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (4)$$

where $\varepsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. Throughout, we say that the two sectors are (gross) *substitutes* when $\varepsilon > 1$ and (gross) *complements* when $\varepsilon < 1$.¹⁴ The case of substitutes $\varepsilon > 1$ (in fact, and elasticity of substitution significantly greater than 1) appears as the more empirically relevant benchmark, since we would expect successful clean technologies to substitute for the functions of dirty technologies. Nevertheless, since the relevant elasticity of substitution has not yet been carefully estimated, and because

¹³ Alternatively, the negative consequences of environmental degradation could have been incorporated into the production structure with equivalent results.

¹⁴ As mentioned in the Introduction, renewable energy that can be stored and transported efficiently would correspond to a high degree of substitution between dirty and clean inputs, since the same production services can be obtained from alternative energy with less pollution. Similarly, cars using gasoline versus cars using clean energy sources would be examples of highly substitutable dirty and clean inputs. In contrast, if “clean alternatives” involved reductions in our consumption of energy or transportation services, this would correspond to a low degree of substitution. Similarly, if “green cars” were produced using components that are required other dirty inputs, the relevant elasticity of substitutability between clean and dirty sectors would be smaller.

the case of complements both highlights a variety of different and novel economic forces and is theoretically interesting, throughout we discuss both cases, though we place more emphasis on the case of substitutes.

Both Y_{ct} and Y_{dt} are produced using labor and a continuum of sector-specific machines (intermediates) according to the production functions

$$Y_{ct} = L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} x_{cit}^\alpha di \text{ and } Y_{dt} = L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} x_{dit}^\alpha di, \quad (5)$$

where $\alpha \in (0, 1)$, A_{jit} is the quality of machine of type i used in sector $j \in \{c, d\}$ at time t and x_{jit} is the quantity of this machine. This setup is similar to Acemoglu (1998), except that employment in the two sectors is endogenously determined and the distribution parameters have been dropped in (4) to simplify the algebra. We also define

$$A_{jt} \equiv \int_0^1 A_{jit} di \quad (6)$$

as the aggregate productivity in sector $j \in \{c, d\}$.

Market clearing for labor requires that

$$L_{ct} + L_{dt} \leq 1. \quad (7)$$

In line with the literature on endogenous technical change, machines (for both sectors) are supplied by monopolistically competitive firms. Regardless of the quality of machines and of the sector for which they are designed, producing one unit of any machine costs ψ units of the final good. Without loss of generality, we normalize $\psi \equiv \alpha^2$.

We also normalize the measure of scientists s to 1. The technology of innovation is as follows. At the beginning of every period, each scientist decides whether to direct her research to clean or dirty technology. She is then randomly allocated to at most one machine (without any congestion; so that each machine is also allocated to at most one scientist) and is successful in innovation with probability $\eta_j \in (0, 1)$ in sector $j \in \{c, d\}$, where innovation increases the quality of a machine by a factor $1 + \gamma$ (with $\gamma > 0$, i.e., from A_{jit} to $(1 + \gamma)A_{jit}$).¹⁵ A successful scientist (who has invented a better version of machine i in sector $j \in \{c, d\}$) obtains a one-period patent and becomes the entrepreneur for the current period in the production of machine i . In sectors where innovation was not successful, monopoly rights are allocated randomly to an entrepreneur drawn from a pool of potential entrepreneurs who can use the old

¹⁵Our model therefore imposes that all technical change takes a “factor-augmenting” form, increasing A_{ct} or A_{dt} . In practice, non-factor-augmenting improvements are also possible, though more difficult to incorporate into a growth model. Acemoglu (2007) provides a comprehensive analysis of the endogenous bias of technology in response to changes in factor supplies without restricting productivity improvements to take a factor-augmenting form.

technology.¹⁶ This technology for innovation where scientists can only target a sector (rather than a specific machine) ensures that scientists are allocated across the different machines in a sector.¹⁷ We denote the mass of scientists working on machines in sector $j \in \{c, d\}$ at time t by s_{jt} , and thus market clearing for scientists takes the form

$$s_{ct} + s_{dt} \leq 1. \quad (8)$$

Finally, the quality of the environment, S_t , evolves according to the difference equation.

$$S_{t+1} = \max \left\{ \min \left\{ -\xi Y_{dt} + (1 + \delta) S_t, \bar{S} \right\}, 0 \right\}, \quad (9)$$

where ξ measures the rate of environmental degradation resulting from the production of dirty inputs, and δ is the rate of “environmental regeneration”. Recall that \bar{S} is the maximum level of environmental quality corresponding to zero pollution. This equation introduces the major externality in our model, from the production of the dirty input to environmental degradation. Note that if $S_t = 0$, then S_τ will remain at 0 for all $\tau > t$.

While other papers in the environment literature typically use more detailed descriptions of environmental dynamics, in this paper we take a more “reduced-form” approach and concentrate instead on identifying the new economic forces that arise in the presence of directed technical change. Nevertheless, the above dynamic equation is meant to capture basic features of the real process of environmental change. First, we assume an exponential regeneration rate δ because greater environmental degradation is typically presumed to lower the regeneration capacity of the globe. For example, part of the carbon in the atmosphere is absorbed by the ice cap; as the ice cap melts because of global warming, more carbon is released into the atmosphere and the melting of the ice cap decreases the albedo of the planet further contributing to global warming. Similarly, the depletion of forests reduces carbon absorption, contributing further to global warming. Second, as already mentioned above, the upper bound \bar{S} captures the idea that environmental degradation results from pollution, and that pollution cannot be negative. We discuss below how our results change if there is no upper bound on environmental quality (i.e., $\bar{S} = \infty$) and under alternative laws of motion for the quality of the environment.

¹⁶ As highlighted further by equation (15) below, this structure implies that innovation builds on the existing level of quality of a machine, and thus incorporates the “building on the shoulders of giants” feature. In terms of the framework in Acemoglu (2002), this implies that there is “state dependence” in the innovation possibilities frontier, in the sense that advances in one sector make future advances in that sector more profitable or more effective. This is a natural feature in the current context, since improvements in fossil fuel technology should not (and in practice do not) directly translate into innovations in alternative and renewable energy sources. Nevertheless, one could allow some spillovers between the two sectors, that is, “limited state dependence” as in Acemoglu (2002), and this would not change our qualitative results.

¹⁷ The assumptions here are adopted to simplify the exposition and mimic the structure of equilibrium in continuous time models (see, for example, Acemoglu, 2002 or Aghion and Howitt, 2009). We adopt a discrete time setup throughout to simplify the analysis of dynamics. Appendix J shows that the qualitative results are identical in an alternative formulation with patents and free entry (instead of monopoly rights being allocated to entrepreneurs).

2.2 Laissez-faire equilibrium

In this subsection we characterize the *laissez-faire equilibrium* outcome, that is, the decentralized equilibrium without any policy intervention. We first characterize the equilibrium production and labor decisions for given productivity parameters. We then analyze the direction of technical change.

An *equilibrium* is given by sequences of wages (w_t), prices for inputs (p_{jt}), prices for machines (p_{jit}), demands for machines (x_{jit}), demands for inputs (Y_{jt}), labor demands (L_{jt}) by input producers $j \in \{c, d\}$, research allocations (s_{dt}, s_{ct}), and quality of environment (S_t) such that, in each period t : (i) (p_{jit}, x_{jit}) maximizes profits by the producer of machine i in sector j ; (ii) L_{jt} maximizes profits by producers of input j ; (iii) Y_{jt} maximizes the profits of final good producers; (iv) (s_{dt}, s_{ct}) maximizes the expected profit of a researcher at date t ; (v) the wage w_t and the prices p_{jt} clear the labor and input markets respectively; and (vi) the evolution of S_t is given by (9).

To simplify the algebra and the notation, we define $\varphi \equiv (1 - \alpha)(1 - \varepsilon)$ and impose the following assumption, which is adopted throughout the text (often without explicitly specifying it).

Assumption 1

$$\frac{A_{c0}}{A_{d0}} < \min \left((1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}}, (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} \right).$$

This assumption imposes the reasonable condition that initially the clean sector is sufficiently backward relative to the dirty (fossil fuel) sector, so that under laissez-faire and with $\varepsilon > 1$, the economy starts innovating in the dirty sector. This assumption enables us to focus on the more relevant part of the parameter space (see Appendix A for the case in which this assumption does not hold).

2.2.1 Laissez-faire equilibrium given technologies

We first consider the equilibrium at time t given technology levels A_{cit} and A_{dit} . For this particular part we drop the subscripts t .

As the final good is produced competitively the ratio of relative price satisfies

$$\frac{p_c}{p_d} = \left(\frac{Y_c}{Y_d} \right)^{-\frac{1}{\varepsilon}}. \quad (10)$$

This equation implies that the relative price of clean inputs (compared to dirty inputs) is decreasing in their relative supply, and moreover, the elasticity of the relative price response is the inverse of the elasticity of substitution between the two inputs. Our normalization of the

final good price at 1 then also implies that

$$[p_c^{1-\varepsilon} + p_d^{1-\varepsilon}]^{1/(1-\varepsilon)} = 1.$$

The profit-maximization problem of the producer of machine i in sector $j \in \{c, d\}$ can be written as

$$\max_{x_{ji}, L_j} \left\{ p_j L_j^{1-\alpha} \int_0^1 A_{ji}^{1-\alpha} x_{ji}^\alpha di - w L_j - \int_0^1 p_{ji} x_{ji} di \right\},$$

and leads to the following iso-elastic inverse demand curve:

$$x_{ji} = \left(\frac{\alpha p_j}{p_{ji}} \right)^{-\frac{1}{1-\alpha}} A_{ji} L_{ji}. \quad (11)$$

Thus the demand for machines i in sector j increases with the price p_j of input j and with employment L_j in that sector, since both increase the profitability of all machines used in that sector, encouraging producers to use more of each. It is also increasing in the quality of such machines, A_{ji} , and decreasing in their price, p_{ji} .

The monopolist producer of machine i in sector j chooses p_{ji} and x_{ji} so as to maximize profits $\pi_{ji} = (p_{ji} - \psi) x_{ji}$, subject to the inverse demand curve (11). Given this iso-elastic demand, the profit-maximizing price is a constant markup over marginal cost, thus $p_{ji} = \psi/\alpha$. Recalling the normalization $\psi \equiv \alpha^2$, this implies that $p_{ji} = \alpha$ and thus the equilibrium demand for machines i in sector j is obtained as

$$x_{ji} = p_j^{\frac{1}{1-\alpha}} L_j A_{ji}. \quad (12)$$

Equilibrium profits for the monopolist are then given by

$$\pi_{ji} = (1 - \alpha) \alpha p_j^{\frac{1}{1-\alpha}} L_j A_{ji}. \quad (13)$$

Next combining equation (12) with the first-order condition with respect to labor, $(1 - \alpha) p_j L_j^{-\alpha} \int_0^1 A_{ji}^{1-\alpha} x_{ji}^\alpha di = w$ and using (6) gives the relative prices of clean and dirty inputs as

$$\frac{p_c}{p_d} = \left(\frac{A_c}{A_d} \right)^{-(1-\alpha)}. \quad (14)$$

This equation formalizes the natural idea that the input produced with more productive machines will be relatively cheaper.

2.2.2 Laissez-faire equilibrium: directed innovation

We next endogeneize productivity by linking productivity growth to R&D in clean and dirty technologies (for clarity, we now reintroduce the time subscripts t).

If a scientist succeeds in innovation, she discovers a new machine that is $(1 + \gamma)$ times more productive than its previous vintage, A_{jit-1} . Therefore, denoting the mass of scientists

directing their effort to sector j by s_{jt} , and recalling that scientists targeting sector j are randomly allocated across machines in that sector, the average productivity in sector j at time t evolves over time according to the difference equation

$$A_{jt} = (1 + \gamma \eta_j s_{jt}) A_{jt-1}. \quad (15)$$

To determine the evolution of average productivities in the two sectors, we need to characterize the profitability of research in these sectors, which will determine the direction of technical change. Taking into account the probability of success, the expected profit Π_{jt} for a scientist engaging in research in sector j is

$$\begin{aligned} \Pi_{jt} &= \eta_j \int_0^1 (1 - \alpha) \alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} (1 + \gamma) A_{jit-1} di \\ &= \eta_j (1 + \gamma) (1 - \alpha) \alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jt-1}, \end{aligned} \quad (16)$$

where the second line simply uses (6). Consequently, the relative benefit from undertaking research in sector c relative to sector d is governed by the ratio:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \times \underbrace{\left(\frac{p_{ct}}{p_{dt}} \right)^{\frac{1}{1-\alpha}}}_{\text{price effect}} \times \underbrace{\frac{L_{ct}}{L_{dt}}}_{\text{market size effect}} \times \underbrace{\frac{A_{ct-1}}{A_{dt-1}}}_{\text{direct productivity effect}}. \quad (17)$$

When this ratio is higher, R&D directed towards the clean technologies becomes more profitable.

Thus a scientist's incentive to innovate in the clean versus the dirty sector machines is shaped by three forces: (i) the *direct productivity effect* (captured by the term A_{ct}/A_{dt}), which pushes towards innovating in the sector with higher productivity; this force results from the presence of the “building on the shoulders of giants” effect highlighted in (15); (ii) the *price effect* (captured by the term $(p_{ct}/p_{dt})^{\frac{1}{1-\alpha}}$), encouraging innovation towards the sector with higher prices, which from (14) is the relatively backward sector; (iii) the *market size effect* (captured by the term L_{ct}/L_{dt}), encouraging innovation in the sector with greater employment, which has the larger market for machines. Which sector has greater employment and a larger market is in turn determined by relative productivities and the elasticity of substitution between the two inputs. The more substitutable the two inputs are, the more important is the market size effect compared to the price effect.

Next, combining (12) with (5), gives the equilibrium production level of input j as

$$Y_{jt} = (p_{jt})^{\frac{\alpha}{1-\alpha}} A_{jt} L_{jt}. \quad (18)$$

Now combining (18) with (10), then using (14) and the definition of $\varphi \equiv (1 - \alpha)(1 - \varepsilon)$, we obtain the relationship between relative productivities and relative employment as:

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{p_{ct}}{p_{dt}} \right)^{-\frac{\varphi-1}{1-\alpha}} \frac{A_{dt}}{A_{ct}} = \left(\frac{A_{ct}}{A_{dt}} \right)^{-\varphi}. \quad (19)$$

Thus the market size effect creates a force towards innovation in the more backward sector when $\varepsilon < 1$, and in the more advanced sector when $\varepsilon > 1$. More specifically, combining (14), (17) and (19), we obtain

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} \right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi}, \quad (20)$$

which yields the following lemma:

Lemma 1 *In the laissez-faire equilibrium, innovation at time t occurs in the clean sector only when $\eta_c A_{ct-1}^{-\varphi} > \eta_d (1 + \gamma\eta_c)^{\varphi+1} A_{dt-1}^{-\varphi}$, in the dirty sector only when $\eta_c (1 + \gamma\eta_d)^{\varphi+1} A_{ct-1}^{-\varphi} < \eta_d A_{dt-1}^{-\varphi}$, and can occur in both sectors when $\eta_c (1 + \gamma\eta_d s_{dt})^{\varphi+1} A_{ct-1}^{-\varphi} = \eta_d (1 + \gamma\eta_c s_{ct})^{\varphi+1} A_{dt-1}^{-\varphi}$.*

Proof. See Appendix A, where we also present a complete characterization of the equilibrium allocations of scientists and equilibrium innovation. ■

The notable conclusion of this lemma is that innovation will favor the more advanced sector when $\varepsilon > 1$. In particular, in this case $\varphi \equiv (1 - \alpha)(1 - \varepsilon) < 0$, and the direct productivity and market size effects are stronger than the price effect. In contrast, when $\varepsilon < 1$, innovation will favor the less advanced sector because $\varphi > 0$ (and the direct productivity effect is weaker than the price effect and the market size effect, which now reinforce each other). Using Lemma 1 we can establish:

Proposition 1 *Suppose Assumption 1 holds. Then there exists a unique laissez-faire equilibrium, which takes the following form:*

- *If $\varepsilon > 1$, innovation always occurs in the dirty sector only.*
- *If $\varepsilon < 1$, innovation first occurs in the clean sector and subsequently occurs in both sectors. The share of scientists devoted to the clean sector in the long run is $s_c = \eta_d / (\eta_c + \eta_d)$.*

Proof. See Appendix B. ■

The intuition for this proposition follows from Lemma 1. When the two inputs are substitutes ($\varepsilon > 1$), innovation starts in the dirty sector, which is more advanced initially (Assumption 1). This increases the gap between the dirty and the clean sectors, and the initial pattern of equilibrium is reinforced. In contrast, in the empirically less relevant but theoretically interesting case where the two inputs are complements ($\varepsilon < 1$), the price effect dominates and innovation initially takes place in the more backward—in this case, the clean—sector. This reduces the technology gap between the two sectors and ultimately the equilibrium must involve innovation in both sectors; in particular, the share of scientists allocated to the clean sector converges towards $s_c = \eta_c / (\eta_c + \eta_d)$, which ensures that both sectors grow at the same rate (see Appendix B). In particular, in this case average quality levels in both sectors, A_c and A_d , grow at the same asymptotic rate $\gamma\tilde{\eta}$, where $\tilde{\eta} \equiv \eta_c \eta_d / (\eta_c + \eta_d)$ —see next subsection.

2.3 Directed technical change and environmental disasters

A major concern by climate scientists is that the environment may deteriorate so much that it reaches a “point of no return”. In our environment equation (9), this notion is captured by the fact that if environmental quality S_t reaches 0 in finite time, it remains at 0 forever thereafter. Motivated by this feature, we define the notion of *environmental disaster*, which will be useful for developing the main intuitions implied by our framework, before we provide a more complete characterization of optimal environmental policy.

Definition 1 *An environmental disaster occurs if $S_t = 0$ for some $t < \infty$.*

Our assumptions on the utility function, in particular, that $u(C, 0) = -\infty$, imply that an environmental disaster cannot be part of a welfare-maximizing allocation (for any $\rho < \infty$). In this subsection, we focus on how a simple policy of “redirecting technical change” can avoid an environmental disaster (when it would otherwise take place in the laissez-faire equilibrium). We will then highlight the role of directed technical change by comparing the results to a model in which scientists cannot direct their research to different sectors.

2.3.1 Disaster in the laissez-faire equilibrium

Output of the two inputs and the final good in the laissez-faire equilibrium can be written as (again dropping time subscripts to simplify notation):

$$\begin{aligned} Y_{ct} &= (A_{ct}^\varphi + A_{dt}^\varphi)^{-\frac{\alpha+\varphi}{\varphi}} A_{ct} A_{dt}^{\alpha+\varphi}, & Y_{dt} &= (A_{ct}^\varphi + A_{dt}^\varphi)^{-\frac{\alpha+\varphi}{\varphi}} A_{ct}^{\alpha+\varphi} A_{dt}, \\ \text{and } Y_t &= (A_{ct}^\varphi + A_{dt}^\varphi)^{-\frac{1}{\varphi}} A_{ct} A_{dt}. \end{aligned} \quad (21)$$

These expressions, together with Proposition 1, imply that in the long-run, dirty input production will grow without bound (when $\varepsilon > 1$, the long-run growth rate of dirty input production is $\gamma\eta_d$, while it is $\gamma\tilde{\eta}$ when $\varepsilon < 1$). As a level of production of dirty input greater than $(1 + \delta)\xi^{-1}\bar{S}$ necessarily leads to a disaster next period, the economy under laissez-faire will eventually reach a disaster. This establishes (proof omitted):

Proposition 2 *Suppose Assumption 1 holds. Then the laissez-faire equilibrium always leads to an environmental disaster.*

Although the laissez-faire equilibrium always involves environmental disaster, the expressions in (21) imply that the long-run growth rate of dirty input production in the substitutable case is $\gamma\eta_d$, which is greater than its long-run growth rate in the complementarity case, $\gamma\tilde{\eta}$, since in the latter case R&D resources are spread across the two sectors. Exploiting this fact, we can show that for given initial technological levels A_{c0} and A_{d0} , the production of dirty input is always higher in the substitutability case than in the complementarity case, which in turn

implies that the disaster occurs sooner in the substitutability case than in the complementarity case:

Corollary 1 *Starting with $S_0 = \bar{S}$, an environmental disaster under laissez-faire takes place sooner when $\varepsilon > 1$ than when $\varepsilon < 1$.*

Proof. See Appendix C. ■

2.3.2 Preventing an environmental disaster using simple policies

Proposition 2 implies that some type of intervention is necessary to avoid a disaster. Suppose first that the government can impose a profit tax q_t on dirty input production, with the proceeds being redistributed lump-sum to the representative household (which is equivalent to a subsidy to scientists working on clean inputs with lump-sum taxes). The expected profit from undertaking research in the dirty sector then becomes

$$\Pi_{dt} = (1 - q_t) \eta_d (1 + \gamma) (1 - \alpha) \alpha p_{dt}^{\frac{1}{1-\alpha}} L_{dt} A_{dt-1},$$

while Π_{ct} is still given by (16). This immediately implies that a sufficiently high profit tax can divert innovation away from the dirty sector.¹⁸ Moreover, while this tax is implemented, the ratio A_{ct}/A_{dt} will grow at the rate $\gamma\eta_c$. The implications of the tax then depend on the degree of substitutability between the two inputs.

When the two inputs are substitutes ($\varepsilon > 1$), a temporary profit tax (maintained for the necessary number of periods, D) is sufficient to redirect all research to the clean sector. More specifically, while the profit tax is being implemented, the ratio A_c/A_d will increase, and when it has become sufficiently high, it will be profitable for scientists to direct their research to the clean sector even without the tax.¹⁹ Then (21) implies that Y_d will grow asymptotically at the same rate as $A_c^{\alpha+\varphi}$. In particular, if $\varepsilon \geq 1/(1 - \alpha)$ (or $\alpha + \varphi \leq 0$), Y_d will not grow in the long-run and thus, as long as the initial environmental quality is sufficiently high, a temporary profit tax policy will be sufficient to avoid an environmental disaster. In contrast, if $\varepsilon \in (1, 1/(1 - \alpha))$ (or $\alpha + \varphi > 0$), equation (21) implies that even after all research is directed

¹⁸In particular, following the analysis in Appendix A, to implement a unique equilibrium where all scientists direct their research to the clean sector, the profit tax q_t must satisfy

$$q_t > 1 - (1 + \gamma\eta_d)^{\varphi+1} \frac{\eta_c}{\eta_d} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} \quad \text{if } \varepsilon \geq \frac{2-\alpha}{1-\alpha} \quad \text{and} \quad q_t \geq 1 - (1 + \gamma\eta_c)^{-(\varphi+1)} \frac{\eta_c}{\eta_d} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} \quad \text{if } \varepsilon < \frac{2-\alpha}{1-\alpha}.$$

¹⁹In particular, the temporary tax needs to be imposed for D periods where D is the smallest integer such that:

$$\frac{A_{ct+D-1}}{A_{dt+D-1}} > (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} \quad \text{if } \varepsilon \geq \frac{2-\alpha}{1-\alpha} \quad \text{and} \quad \frac{A_{ct+D-1}}{A_{dt+D-1}} \geq (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} \quad \text{if } 1 < \varepsilon < \frac{2-\alpha}{1-\alpha}.$$

to the clean sector, Y_d will keep growing at rate $(1 + \gamma\eta_c)^{\alpha+\varphi} - 1$. The contrast between these two (strong and weak) substitutability cases is important but also intuitive: first, since $\varepsilon > 1$, as the average quality of clean machines increases, workers get reallocated towards the clean sector (which is just the market size effect); but at the same time the increase of the relative price of the dirty input over time encourages production of the dirty input (price effect). Which of these two forces dominates will depend upon whether $\varepsilon > 1/(1 - \alpha)$ or $\varepsilon < 1/(1 - \alpha)$.²⁰ In the latter case, a temporary tax policy redirecting research to the clean sector will not be sufficient to avoid an environmental disaster.

When the two inputs are complements ($\varepsilon < 1$), with or without a temporary profit tax, the more backward sector always catches up with the more advanced sector in the long run.²¹ Thus in the long run, innovation will take place in both sectors, and production of the dirty input will grow without limit, and consequently, an environmental disaster becomes unavoidable.

This discussion establishes the following proposition (proof in the text):

Proposition 3 *When the two inputs are strong substitutes ($\varepsilon > 1/(1 - \alpha)$) and \bar{S} is sufficiently high, a temporary profit tax will prevent an environmental disaster. In contrast, when the two inputs are complements or weak substitute ($\varepsilon < 1/(1 - \alpha)$), a profit tax cannot prevent an environmental disaster.*

Thus, when the two inputs are *strong* substitutes, *redirecting* technical change using a temporary policy intervention can be sufficient to avoid a disaster. This shows the importance of directed technical change: temporary incentives are sufficient to induce research to be directed to clean technologies, and once clean technologies are sufficiently advanced, innovation and production will shift sufficiently towards those technologies that environmental disaster can be avoided without further intervention.

Nevertheless, the policy intervention still has economic costs because during the period of adjustment (while productivity in the clean sector is catching up with that in the dirty sector), final output increases more slowly than had innovation been directed towards the dirty sector. We will study the welfare costs of intervention in subsection 2.4. Before doing this, it is instructive to look at a simple measure of the (short-run) cost of intervention, defined as the number of periods T necessary for the economy under the policy intervention to reach the same

²⁰The case where $\varepsilon \in (1, 1/(1 - \alpha))$ is interesting because even though a simple intervention can ensure that all research is directed to clean technologies, this is not sufficient to prevent growth of the production of dirty inputs. This is because the rise in the price of dirty inputs encourages an increase in their production by increasing the use of machines (recall equation (12)). A slightly different intuition for this result is that improvements in the technology of the clean sector also correspond to improvements in the technology of the final good, which uses them as inputs; the final good, in turn, is an input for the dirty sector because machines employed in the sector are produced using the final good.

²¹The proof of this claim follows closely the proof of Proposition 1. In particular, regardless of which sector innovation is first directed at, innovation in the long run must take place in both sectors, which in turn implies that the long-run growth rate must be $\gamma\tilde{\eta}$.

level of output as it would have done within one period in the absence of the intervention: in other words, this is the length of the transition period or the number of periods of “slow growth” in output growth. This measure T (starting at time t) is then the smallest integer such that:

$$\frac{(1 + \gamma\eta_c)^T}{\left((1 + \gamma\eta_c)^{T\varphi} A_{ct-1}^\varphi + A_{dt-1}^\varphi\right)^{\frac{1}{\varphi}}} \geq \frac{(1 + \gamma\eta_d)}{\left(A_{ct-1}^\varphi + (1 + \gamma\eta_d)^\varphi A_{dt-1}^\varphi\right)^{\frac{1}{\varphi}}}$$

or equivalently,

$$T = \left\lceil \frac{\ln \left(((1 + \gamma\eta_d)^{-\varphi} - 1) \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^\varphi + 1 \right)}{-\varphi \ln(1 + \gamma\eta_c)} \right\rceil \quad (22)$$

It can be verified that starting at any $t \geq 1$ along the equilibrium path in Proposition 1, we have $T \geq 1$. Thus once innovation is directed towards the clean sector it will take more than one period for the economy to achieve the same output growth as it would have achieved in just one period in the equilibrium of Proposition 1 (with innovation still directed at the dirty sector). The following corollary then follows immediately from equation (22), in particular, recalling that $\varphi \equiv (1 - \alpha)(1 - \varepsilon)$ (proof omitted):

Corollary 2 *For $A_{dt-1}/A_{ct-1} \geq 1$, the short-run cost of intervention, T , is nondecreasing in the technology gap A_{dt-1}/A_{ct-1} and the elasticity of substitution ε . Moreover, T increases more with A_{dt-1}/A_{ct-1} when ε is greater.*

The (short-run) cost of intervention, T , is increasing in A_{dt-1}/A_{ct-1} because a larger gap between the initial quality of dirty and clean machines leads to a longer transition phase, and thus to a longer period of low growth. In addition, T is also increasing in the elasticity of substitution ε . Intuitively, if the two inputs are close substitutes, final output production relies mostly on the more productive input, and therefore, productivity improvement in the clean sector (taking place during the transition phase) will have less impact on overall productivity until the clean technologies surpass the dirty ones.

The corollary shows that layering intervention is costly, not only because of the continued environmental degradation that will result, but also because during the period of delay A_{dt}/A_{ct} will increase further, and thus when the intervention is eventually implemented, the temporary profit tax will need to be imposed for longer and there will be a longer period of slow growth (higher T). This result is clearly related to the “building on the shoulders of giants” feature of the innovation process. Furthermore, the result that the effects of ε and A_{dt-1}/A_{ct-1} on T are complementary implies that delaying the starting date of the intervention is more costly when the two inputs are more substitutable.

We next illustrate the additional costs created by delayed intervention by computing the value of T for different values of the elasticity of substitution ε (in all cases we take ε greater

than $1/(1 - \alpha)$) and different starting dates of intervention.²² We chose the parameters of the model as follows: we set $\eta_c = \eta_d = 0.025$ and $\gamma = 1$ so that the long-run growth rate $\gamma\eta_j$ is equal to 2.5% (and therefore time periods correspond to years); we take $\alpha = 1/3$ (so that the share of national income spent on machines is approximately equal to the share of capital). We then compute A_{ct-1} and A_{dt-1} so as that the implied values of Y_{ct-1} and Y_{dt-1} match the 2006 production of nonfossil and fossil fuel in the world primary energy supply (respectively, 19.2% and 80.8% of a total of 11714 Mtoe, International Energy Agency, 2008). Note that in our exercises here and below, when ε varies, A_{ct-1} and A_{dt-1} also need to be adjusted (in particular, a higher ε leads to a higher value of the ratio A_{ct-1}/A_{dt-1}).²³

Using these parameter values, Table 1 shows that T increases rapidly with the delay in intervention and/or the substitutability between the two inputs (even though as we increase ε , we also increase the initial ratio A_c/A_d). Moreover, the increase in T in response to delay is itself increasing in ε .

Table 1: The Cost of Delayed Intervention

(Values of T as a function of delay and ε)

delay\ ε	2	3	5	10
0	3	3	3	4
5	8	8	9	11
10	13	14	16	19
20	23	25	29	37
30	34	37	45	57

Overall, the analysis in this subsection has established that a simple policy intervention that consists on “redirecting” technical change towards environment friendly technologies can help prevent an environmental disaster. Our analysis also highlights the idea that delaying intervention may be quite costly, not only because it further damages the environment (an effect already recognized in the climate science literature), but also because it widens the gap between the dirty and clean technologies, thereby inducing a longer period of catch-up with slower growth.

2.3.3 Comparison to undirected directed technical change

Here we briefly consider a variant of the model studied so far but without directed technical change. Our purpose is to highlight how the endogeneity of the direction of technical change is crucial for the effects of temporary policy interventions. In particular, consider the same

²²We discuss the cost of delayed intervention more systematically in terms of welfare losses in the next section.

²³The other parameters of the model do not enter equation (22), hence we do not specify them until the somewhat more detailed calibration exercise in subsection 2.4.3.

environment as before, but now suppose that scientists are randomly allocated across sectors so as to ensure equal growth in the qualities of clean and dirty machines (at the rate $\gamma\tilde{\eta}$). This implies that a fixed fraction $s_c = \eta_d / (\eta_c + \eta_d)$ of scientists are always allocated to the clean sector.²⁴ Consequently, the production of dirty input will always grow at rate $\gamma\tilde{\eta}$, so that an environmental disaster will always occur, with or without temporary profit tax.

Note however that in the case where the two inputs are strong substitutes ($\varepsilon > 1/(1 - \alpha)$), under laissez-faire a disaster will occur sooner with directed technical change than without. This in turn follows from the fact that for given initial technologies on clean and dirty machines, all innovation is directed towards the dirty sector. Yet, when the two inputs are strong substitutes, we also know that with directed technical change a temporary profit tax can redirect innovations towards the clean sector, thereby preventing an environmental disaster, whereas it cannot prevent a disaster when technical change is “undirected”. This establishes the following proposition further demonstrating the importance of directed technical change in our results (proof in the text):

Proposition 4 *When $\varepsilon > 1/(1 - \alpha)$, an environmental disaster under laissez-faire arises earlier with directed technical change than in the equivalent economy with undirected technical change. However, provided that \bar{S} is sufficiently high, a temporary profit tax can prevent an environmental disaster with directed technical change but not in the equivalent economy with undirected technical change.*

2.3.4 Alternative laws of motion for environmental quality

The directed technical change ideas can be applied with alternative laws of motion. One possibility is to modify (9) so that there is no upper bound to environmental quality, i.e., to set $\bar{S} = \infty$ (and set the initial environmental quality at some finite level $S_0 < \infty$). This implies that a disaster can be avoided even if dirty input production grows at a positive rate, provided that this rate is lower than the regeneration rate of the environment, δ . Recall that under laissez-faire the production of dirty input grows at a rate $\gamma\eta_d$ if $\varepsilon > 1$ and at a rate $\gamma\tilde{\eta}$ if $\varepsilon < 1$; moreover, a temporary profit tax can still permanently divert innovation towards the clean sector when the two inputs are substitutes, so that in the long-run dirty input production decreases in the strong substitutability case ($\varepsilon > 1/(1 - \alpha)$) and increases at a rate $(1 + \gamma\eta_c)^{\alpha+\varphi} - 1$ in the weak substitutability case ($1 < \varepsilon < 1/(1 - \alpha)$). Consequently, in this case Proposition 2 and 3 are replaced by the following proposition (proof omitted):

Proposition 5 *Suppose that $\bar{S} = \infty$. Then, under laissez-faire, there will be an environmental disaster if the regeneration rate of the environment, δ , is sufficiently low, in particular, if $\delta < \gamma\eta_d$ when $\varepsilon > 1$, and if $\delta < \gamma\tilde{\eta}$ when $\varepsilon < 1$.*

²⁴If we instead assumed that half of the scientists are allocated to the clean sector, the qualitative results would be similar, though the expressions would become somewhat more complicated.

When the two inputs are strong substitutes ($\varepsilon > 1/(1 - \alpha)$) and the initial environmental quality, S_0 , is sufficiently high, a temporary profit tax is sufficient to prevent a disaster. When the two inputs are weak substitutes ($1/(1 - \alpha) > \varepsilon > 1$) and the initial environmental quality, S_0 , is sufficiently high, a temporary profit tax can prevent an environmental disaster only if $(1 + \gamma\eta_c)^{\alpha+\varphi} - 1 < \delta$. When the two inputs are complements ($\varepsilon < 1$), a temporary profit tax cannot prevent an environmental disaster (i.e., if $\delta < \gamma\tilde{\eta}$, an environmental disaster will take place both under laissez-faire and under a temporary profit tax).

It is also interesting to repeat the comparison to an environment with undirected technical change with this modified law of motion. This comparison is also straightforward: recall that with undirected technical change and $\varepsilon > 1$, the laissez-faire growth rate of the production of the dirty input is $\gamma\tilde{\eta}$, which is smaller than its long-run growth rate $\gamma\eta_d$ with directed technical change. This in turn implies that when $\varepsilon > 1$ the range of δ under which a disaster occurs, for any initial environmental quality, is larger with directed technical change than without. The following proposition is the analog of Proposition 4 and states this result (proof omitted):

Proposition 6 *Suppose that $\bar{S} = \infty$. Then, if $\varepsilon < 1$, the range of δ for which an environmental disaster occurs for any initial environmental quality is the same with or without a profit tax (regardless of whether there is directed technical change). If $\varepsilon > 1/(1 - \alpha)$, the range of δ for which a disaster occurs under laissez-faire for any initial environmental quality is greater with directed technical change than without (δ must be less than $\gamma\eta_d$ with directed technical change, whereas it must be less than $\gamma\tilde{\eta}$ without directed technical change). However, a temporary profit tax makes this range become smaller with directed technical change (and prevents an environmental disaster for sufficiently high initial quality when $\varepsilon > 1/(1 - \alpha)$), whereas it has no effect on this range with undirected technical change.*

This proposition therefore shows, perhaps even more clearly than in the case with $\bar{S} < \infty$, that directed technical change is a force towards an environmental disaster under laissez-faire because it encourages more innovation towards the relatively advanced dirty sector.²⁵ However, it also becomes a potential powerful remedy for avoiding a disaster because it allows redirecting technical change when the government uses simple policy interventions.

²⁵This proposition does not cover the case in which $1/(1 - \alpha) > \varepsilon > 1$, since with this parameter configuration, the comparison of the likelihood of an environmental disaster with and without directed technical change is ambiguous. In particular, as the preceding discussion makes it clear, with undirected technical change, an environmental disaster takes place with or without temporary policy intervention if $\gamma\tilde{\eta} > \delta$. In contrast, with directed technical change, an environmental disaster takes place without policy intervention for the larger set of parameter values $\gamma\eta_d > \delta$; but with policy intervention (and a sufficiently large initial stock of environmental quality), it can be prevented if $(1 + \gamma\eta_c)^{\alpha+\varphi} - 1 < \delta$, which could be a smaller or larger set of parameter values than $\gamma\tilde{\eta} < \delta$.

Finally, let us also note that the analysis can also be conducted under another alternative law of motion of environmental quality,

$$S_{t+1} = -\xi Y_{dt} + S_t + \Delta,$$

where regeneration of the environment is additive rather than proportional to current quality. With this alternative law of motion, it is straightforward to show that the results are essentially identical to the baseline formulation because a disaster can only be avoided if Y_{dt} does not grow at a positive exponential rate in the long run. Consequently, in this case, Propositions 2, 3 and 4 continue to apply.

2.4 Optimal environmental policy

We have so far studied the behavior of the laissez-faire equilibrium and discussed how environmental disaster may be avoided. In this subsection, we characterize the optimal allocation of resources in this economy and discuss how it can be decentralized by using profit and input taxes. The socially-planned (optimal) allocation will “correct” for two externalities: (1) the environmental externality exerted by dirty input producers, and (2) the knowledge externalities from R&D (the fact that in the laissez-faire equilibrium scientists do not internalize the effects of their research on productivity in the future). In addition, the planner can and will correct for the standard static monopoly distortion in the price of machines, encouraging more intensive use of existing machines (see, for example, Acemoglu, 2009). Throughout this section, we assume that the social planner (government) has access to lump-sum taxes and transfers to complement the other policy instruments (and thus raise or redistribute revenues as required).

2.4.1 The social planner’s problem

The social planner’s problem is one of choosing a dynamic path of final good production Y_t , consumption C_t , input productions Y_{jt} , expected machines production x_{jit} , labor share allocation L_{jt} , scientists allocation s_{jt} , environmental quality S_t , and quality of machines A_{jit} , that maximizes the intertemporal utility of the representative consumer, (1), subject to (4), (5), (7), (8), (9), (15), and

$$C_t = Y_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right). \quad (23)$$

Let λ_t denote the Lagrange multiplier for (4), which is naturally also the shadow value of one unit of final good production. The first-order conditions with respect to Y_t imply that this shadow value is also equal to the Lagrange multiplier for (23), so that it is also equal to the shadow value of one unit of consumption. Then the first-order condition with respect to C_t

yields

$$\lambda_t = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial C}, \quad (24)$$

so that, again naturally, the shadow value of the final good is equal to the marginal utility of consumption.

The ratio λ_{jt}/λ_t can then be interpreted as the shadow price of input j at time t (relative to the price of the final good). To emphasize this interpretation, we will denote this ratio by p_{jt} . We can now combine the first-order condition with respect to x_{ji} with (5) to obtain:²⁶

$$Y_{jt} = \left(\frac{\alpha}{\psi} p_{jt} \right)^{\frac{\alpha}{1-\alpha}} A_{jt} L_{jt} \quad (25)$$

so that for given price, average technology and labor allocation, the production of each input is scaled up by a factor $\alpha^{\frac{\alpha}{1-\alpha}}$ compared to the laissez-faire equilibrium (this results from the more intensive use of machines in the socially-planned allocation).

Next, letting ω_t denote the Lagrange multiplier for the environmental equation (9), the first-order condition with respect to S_t gives

$$\omega_t = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial S} + (1+\delta) I_{S_t < \bar{S}} \omega_{t+1}, \quad (26)$$

where $I_{S_t < \bar{S}}$ is equal to 1 if $S_t < \bar{S}$ and 0 otherwise. This implies that the price of one unit of environmental quality at time t is equal to the marginal utility that it generates in this period plus the price of $(1+\delta)$ units of environmental quality at time $t+1$ (as one unit of environmental quality at time t generates $1+\delta$ units at time $t+1$), so that the price of one unit of environmental quality is equal to the marginal utility this unit generates in all subsequent periods. In particular, this implies that if for all $\tau > T$, $S_\tau = \bar{S}$, then $\omega_t = 0$ for all $t > T$.

The first-order conditions with respect to Y_{ct} and Y_{dt} then give

$$\begin{aligned} Y_{ct}^{\frac{-1}{\varepsilon}} \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} &= p_{ct}, \\ Y_{dt}^{\frac{-1}{\varepsilon}} \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} - \frac{\omega_{t+1}\xi}{\lambda_t} &= p_{dt}. \end{aligned} \quad (27)$$

These equations imply that compared to the laissez-faire equilibrium, the social planner introduces a wedge of $\omega_{t+1}\xi/\lambda_t$ between the marginal product of the dirty input in the production

²⁶This first-order condition with respect to x_{ji} yields

$$x_{jit} = \left(\frac{\alpha}{\psi} p_{jt} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} = \left(\frac{1}{\alpha} p_{jt} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt},$$

which can be compared to the equilibrium inverse demand, (11), and highlights that existing machines will be used more intensively in the socially-planned allocation. This is a natural consequence of the monopoly distortions and can also be interpreted as the socially-planned allocation involving a subsidy of $1-\alpha$ in the use of machines, so that their price should be identical to the marginal cost, i.e., $(1-(1-\alpha))\psi/\alpha = \psi \equiv \alpha^2$.

and its price. This wedge $\omega_{t+1}\xi/\lambda_t$ is equal to the environmental cost of an additional unit of the dirty input (evaluated in terms of units of the final good at time t ; recall that one unit of dirty production at time t destroys ξ units of environmental quality at time $t+1$). Naturally, this wedge is also equivalent to a tax of

$$\tau_t = \frac{\omega_{t+1}\xi}{\lambda_t p_{dt}} \quad (28)$$

on the use of dirty input by the final good producer. This tax rate will be higher when the shadow value of environmental quality is greater, when the marginal utility of consumption today is lower, and when the price of dirty input is lower.²⁷

Finally, the social planner must correct for the knowledge externality. Let μ_{jt} denote the Lagrange multiplier for equation (15) for $j = c, d$. Naturally, this variable would then correspond to the shadow value of average productivity in sector j at time t . The relevant first-order condition then gives:

$$\mu_{jt} = \lambda_t \left(\frac{\alpha}{\psi} \right)^{\frac{1}{1-\alpha}} (1-\alpha) p_{jt}^{\frac{1}{1-\alpha}} L_{jt} + (1 + \gamma \eta_j s_{jt+1}) \mu_{j,t+1}. \quad (29)$$

Intuitively, the shadow value of a unit increase in average productivity in sector $j \in \{c, d\}$ is equal to its marginal contribution to time- t utility plus its shadow value at time $t+1$ times $(1 + \gamma \eta_j s_{jt+1})$ (the number of units of productivity created out of it at time $t+1$). This last term captures the intertemporal knowledge externality.

At the optimum, scientists will be allocated towards the sector with the higher social gain from innovation, as measured by $\gamma \eta_j \mu_{jt} A_{jt-1}$. Using (29), we then have that the social planner will allocate scientists to clean sector whenever the ratio

$$\frac{\eta_c (1 + \gamma \eta_c s_{ct})^{-1} \sum_{\tau \geq t} \lambda_\tau p_{c\tau}^{\frac{1}{1-\alpha}} L_{c\tau} A_{c\tau}}{\eta_d (1 + \gamma \eta_d s_{dt})^{-1} \sum_{\tau \geq t} \lambda_\tau p_{d\tau}^{\frac{1}{1-\alpha}} L_{d\tau} A_{d\tau}} \quad (30)$$

is greater than 1. This contrasts with the decentralized outcome where scientists are allocated according to the private value of innovation, that is, according to the ratio

$(\lambda_t p_{ct}^{\frac{1}{1-\alpha}} L_{ct} A_{ct} / \lambda_t p_{dt}^{\frac{1}{1-\alpha}} L_{dt} A_{dt})$.²⁸ As suggested by the discussion in the text, the optimal en-

²⁷In Appendix D, we show that the tax rates is uniquely defined by

$$\tau_t^{1-\varepsilon} = \left(\frac{\omega_{t+1}\xi}{\lambda_t} \right)^{1-\varepsilon} \left(1 + \left(\frac{A_{dt}^{1-\alpha}}{(1+\tau_t) A_{ct}^{1-\alpha}} \right)^{1-\varepsilon} \right).$$

Using this expression, it is straightforward to establish that this optimal tax rate is increasing in A_{dt}/A_{ct} , from which the last result follows. This expression also shows the first two results more explicitly.

²⁸The knowledge externality is extreme in our model because researchers (scientists) capture profits from innovation for only one period. Nevertheless, a similar externality exists more generally in endogenous and directed technical change models, where researchers do not fully capture the social value of innovation because of both monopoly distortions and knowledge spillovers on future innovations (e.g., Acemoglu, 2002).

vironmental policy can be implemented using a simple tax scheme, as stated in the next proposition.

Proposition 7 *The social planner can implement the social optimum through a tax on the use of the dirty input, a tax/subsidy on the profits realized in the dirty sector and a subsidy for the use of all machines (all proceeds from taxes/subsidies being redistributed/financed lump-sum).*

Proof. See Appendix D. ■

That we need both an input tax and a profit tax to implement the social optimum (in addition to the subsidy to remove the monopoly distortions) is intuitive: the profit tax deals with future environmental externalities by directing innovation and therefore technical progress towards the clean sector, whereas the input tax deals more directly with the current environmental externality by reducing current production of the dirty input, which causes this externality in the first place. By reducing production in the dirty sector, the input tax also discourages innovation in that sector. However, using only the input tax to deal with both current environmental externalities and future (knowledge-based) externalities would necessitate a very high input tax, potentially distorting current production and reducing current consumption excessively. An important implication of this result is therefore that, without additional restrictions on policy, it would not be optimal to rely only on a carbon tax to deal with global warming; one should also use additional instruments (profit tax, R&D subsidies, etc.) that direct innovation towards clean technologies, so that in the future production can be increased using alternative technologies.

2.4.2 The structure of optimal environmental regulation

In subsection 2.3, we showed that a temporary profit tax could prevent a disaster when the two inputs are substitutes. Here we show that, when the two inputs are sufficiently substitutable and the discount rate is sufficiently low, the optimal policy characterized in Proposition 7 also only involves temporary interventions (except for the standard subsidy that correct for monopoly distortions).

More formally, recall that the optimal input tax schedule is given by $\tau_t = \omega_{t+1}\xi/\lambda_t p_{dt}$, where ω_{t+1} , the shadow value of one unit of environmental quality at time $t + 1$, is equal to the discounted marginal utility of environmental quality as of period $t + 1$, that is:²⁹

$$\omega_{t+1} = \sum_{v=t+1}^{\infty} (1 + \delta)^{v-(t+1)} \frac{1}{(1 + \rho)^v} \frac{\partial u(C_v, S_v)}{\partial S} I_{S_{t+1}, \dots, S_v < \bar{S}},$$

²⁹ We must have: $\lim_{t \rightarrow \infty} \omega_t < \infty$, otherwise all ω_t would be infinite.

where $I_{S_{t+1}, \dots, S_\nu < \bar{S}}$ takes value 1 if $S_{t+1}, \dots, S_\nu < \bar{S}$ and 0 otherwise. Thus, using (24), we get

$$\tau_t = \frac{\xi}{p_{dt}} \frac{\frac{1}{1+\rho} \sum_{v=t+1}^{\infty} \left(\frac{1+\delta}{1+\rho} \right)^{v-(t+1)} I_{S_{t+1}, \dots, S_\nu < \bar{S}} \partial u(C_v, S_v) / \partial S}{\partial u(C_t, S_t) / \partial C}. \quad (31)$$

This expression shows that once S_t reaches the upper bound \bar{S} , then the optimal tax on dirty input falls down to zero since $\partial u(C_{t+1}, \bar{S}) / \partial S = 0$. This, in turn, has implications on how the dynamics of the optimal tax schedule depend upon the degree of substitutability between the clean and the dirty inputs.

Proposition 8 *If $\varepsilon > 1/(1 - \alpha)$ and the discount rate ρ is sufficiently small, the optimal input tax, τ_t , and the optimal profit tax, q_t , are both temporary; in the long-run innovation occurs only in the clean sector and the economy grows at a rate $\gamma\eta_c$. If $1 < \varepsilon < 1/(1 - \alpha)$, then in the long-run innovation still only occurs in the clean sector and the economy grows at a rate $\gamma\eta_c$, but the optimal input tax is permanent. Finally, if $\varepsilon < 1$, then the optimal input tax and the optimal profit tax are permanent, and the long-run growth rate is zero.*

Proof. See Appendix E. ■

To obtain an intuition for this proposition, first note that an optimal policy requires avoiding a disaster, since a disaster leads to $u(C, 0) = -\infty$. This in turn implies that the production of dirty input must always remain below a fixed upper bound. This does not prevent positive long-run growth in the substitutability case because final production can increasingly rely on the clean input. An appropriately-chosen profit tax then ensures that innovation occurs only in the clean sector, and when A_c has sufficiently exceeded A_d , innovation in the clean sector will have become sufficiently profitable that it will continue after the profit tax is removed (and hence there is no longer a need for the profit tax). When the two inputs are strong substitutes ($\varepsilon > 1/(1 - \alpha)$), production of dirty input decreases to 0 over time, and as a result, the environmental stock S_t reaches \bar{S} in finite time due to positive regeneration. This in turn ensures that the optimal input tax will reach zero in finite time. Since dirty input production converges to zero, the economy will generate a long-run growth rate equal to the growth rate of A_c , namely $\gamma\eta_c$.

In the complementarity case, the long-run growth rate of final output is the minimum of the long-run growth rates of the two inputs, so it is not possible to achieve positive long-run growth while avoiding a disaster. Nevertheless, avoiding an environmental disaster is still optimal, so optimal environmental regulation will stop long-run growth.

2.4.3 A simple calibration exercise

Propositions 7 and 8 provided insights into the qualitative features of optimal environmental policy. The next step is a careful quantitative analysis to investigate how the endogenous

response of the direction of technical change affects the costs and benefits of different environmental policies. While such a quantitative analysis is beyond the scope of the present paper, we take a first step in this direction by presenting the results of a simple calibration exercise, in particular focusing on the effects of different discount rates and elasticities of substitution on the timing of a switch (of R&D) to clean technology under optimal environmental regulation.

We use the same parameters as in Table 1, with $\eta_c = \eta_d = 0.025$, $\gamma = 1$, $\alpha = 1/3$, and the values of A_c and A_d are computed exactly as described above, to match Y_c and Y_d with nonfossil and fossil fuel consumptions in world energy supply and 2006. Estimating the elasticity of substitution that would be appropriate for this exercise is beyond the scope of our simple calibration exercise here. We simply note that since fossil and nonfossil fuels should be close substitutes (once nonfossil fuels can be transported efficiently), reasonable values of ε should be quite high. Here we start with a moderate baseline case of $\varepsilon = 3$, which is the elasticity between nonfossil and fossil fuel energy used in Van der Zwaan et al (2002), and then also report results for $\varepsilon = 5$ and higher values.

To relate the environmental quality variable S to the atmospheric concentration of carbon, we used a widely-adopted approximation to the relationship between the increase in temperature since preindustrial times (in degrees celsius), Δ , and the atmospheric concentration of carbon dioxide (in ppm), C_{CO_2} :

$$\Delta = 3 \log_2 (C_{CO_2}/280).$$

This equation implies that a doubling of atmospheric concentration in CO_2 (since pre-industrial time, when the concentration was equal 280 ppm) leads to 3°C increase in current temperature (see, e.g., the Fourth Assessment Report of the Intergovernmental Panel on Climate Change). We then express S as a decreasing function of Δ and thus of C_{CO_2} , so that $S = 0$ corresponds to a level of temperature change Δ approximating “disaster temperature” $\Delta_{disaster}$ (described below). More specifically, we set

$$S = 280 \times 2^{\Delta_{disaster}/3} - \max \{C_{CO_2}, 280\}.$$

We relax the assumption that $S_0 = \bar{S}$ and set the initial environmental quality S_0 to correspond to the current atmospheric concentration of 379 ppm (\bar{S} , in turn, corresponds to $C_{CO_2} = 280$ ppm, the pre-industrial value).

We then estimate parameter ξ from the observed value of Y_d and the annual emission of CO_2 (ξY_d in our model) in 2005 (7.5 GTC). Finally, we choose δ such that only half of the amount of emitted carbon contributes to increasing CO_2 concentration in the atmosphere (the rest being offset by “environmental regeneration,” see again the Fourth Assessment Report of the Intergovernmental Panel on Climate Change). This implies a value of $\delta = 0.0009$.

Nordhaus and much of the literature following his work assume that environmental quality affects aggregate productivity. We find it more reasonable that high temperature levels and

high concentrations of carbon dioxide affect utility as well as production and we formulated our model under the assumption that environmental quality directly affects utility. Nevertheless, to highlight the similarities and the differences between our model and existing quantitative models with exogenous technology, we choose the parameters such that the welfare consequences of changes in temperature (for the range of changes observed so far) are the same in our model as in previous work. We parameterize the utility function as

$$u(C_t, S_t) = \frac{(\phi(S_t) C_t)^{1-\sigma}}{1-\sigma}, \quad (32)$$

with $\sigma = 1.4$, which is a CRRA function with a relative risk aversion parameter of 1.4, and contains an additional term $\phi(S)$ capturing the effects of environmental quality. We choose this function as

$$\phi(S) = \varphi(\Delta(S)) \equiv \frac{(\Delta_{disaster} - \Delta(S))^\lambda - \lambda \Delta_{disaster}^{\lambda-1} (\Delta_{disaster} - \Delta(S))}{(1-\lambda) \Delta_{disaster}^\lambda}, \quad (33)$$

for $\Delta(S) \in [0, \Delta_{disaster}]$, where φ is strictly decreasing and concave function, with $\varphi(0) = 1$, $\varphi(\Delta_{disaster}) = 0$, $\varphi'(0) = 0$ and $\lim_{\Delta \rightarrow \Delta_{disaster}} \varphi'(\Delta) = -\infty$, so that our assumptions on the utility function, (2) and (3), are satisfied. In particular, (33) defines a flexible family of continuous functions parameterized by λ . As $\lambda \rightarrow 1$, this function converges to $\varphi_1(\Delta) = (1 - \Delta/\Delta_{disaster})(1 - \ln(1 - \Delta/\Delta_{disaster}))$ for all $\Delta \in [0, \Delta_{disaster})$ (from L' Hopital's rule) and $\varphi_1(\Delta_{disaster}) = 0$, and as $\lambda \rightarrow 0$, it converges (pointwise) to the "step function" $\varphi_0(\Delta) = 1$ for all $\Delta \in [0, \Delta_{disaster})$ and $\varphi_0(\Delta) = 0$ for $\Delta = \Delta_{disaster}$. For our baseline calibration we choose $\Delta_{disaster} = 9.2^\circ\text{C}$, which is twice the highest estimate of the temperature increase that would eventually lead to the melting of the Greenland Ice Sheet (the Fourth Assessment Report of the Intergovernmental Panel on Climate Change). We then compute λ in order to match our function φ with the mapping between temperature and final output in Nordhaus' DICE 2007 model over the range of temperature increases up to 3.5°C . This leads to a value of $\lambda = 0.3933$ in (33), and with this choice of φ function, our model indeed generates effects that are very close to those obtained in Nordhaus's calibration exercises for increases in temperature less than 3.5°C . In our calibrations the temperature increases remain within this range of values under the optimal environmental regulation, however, naturally the implications of temperature increases outside this range influence the structure of optimal policy.³⁰

The debate between Stern and Nordhaus highlighted that the discount rate plays an important role in the implications of models with exogenous technology on optimal environmental policy. We start with a moderate discount rate $\rho = 0.01$ (per annum). Since $\sigma = 1.4$ in our

³⁰Here, we note that in Nordhaus's quantitative exercises, the damage from temperature increases beyond 3.5°C still remain modest. We do not find this feature, which is based on out-of-sample extrapolation, plausible, and in our specification, where environmental quality directly affects utility, the cost of increases above 3.5° , particularly those close to 9.2° , are substantial. It seems very plausible to us that increases in temperature close to 9.2°C would have disastrous consequences for utility (as well as production).

calibration, with a growth rate of 2.5% a year, this discount rate implies a long-run interest rate of $r = \rho + \sigma g = 0.01 + 1.4 \times 0.025 = 4.5\%$.

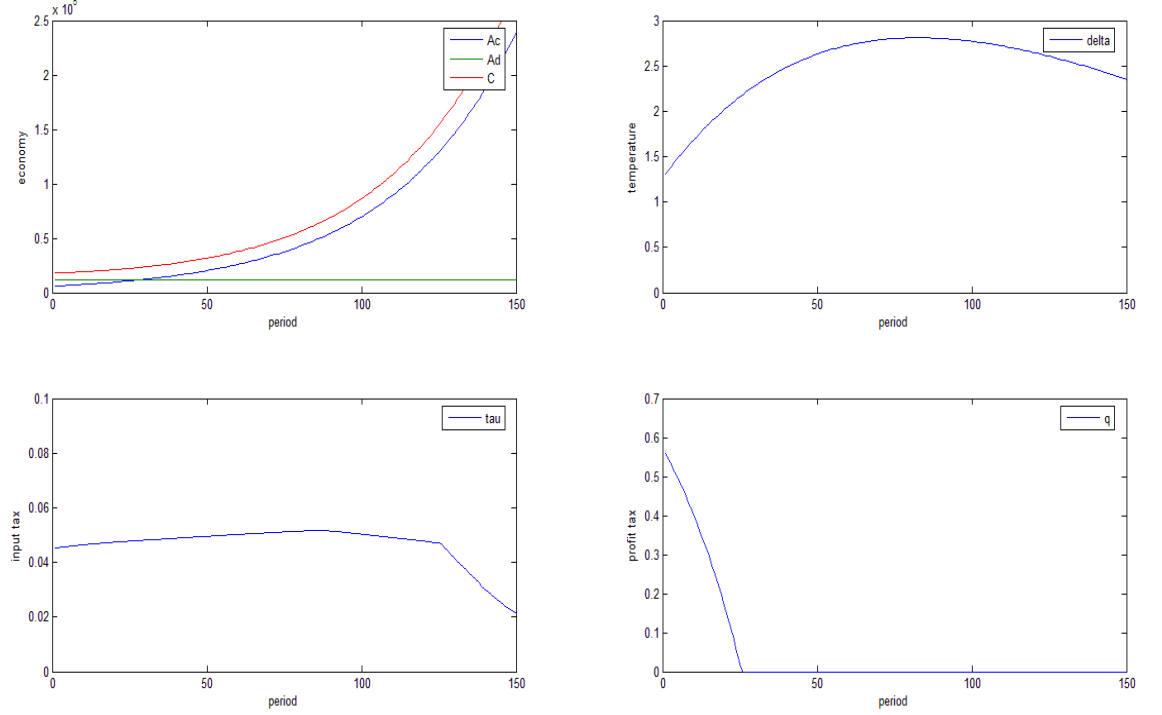


Figure 1: Results from baseline calibration with $\rho = 0.01$ under the optimal environmental regulation. The figure shows the evolution of clean and dirty technologies (A_c and A_d) and aggregate consumption (C) in panel A, of the temperature increase (Δ) in panel B, the optimal input tax (τ) in panel C, and the optimal profit tax (q) in panel D.

Figure 1A shows the form of optimal policy under these parameter choices. It shows that optimal environmental policy induces an immediate (and permanent) switch of all R&D to clean technologies. Consequently, when this optimal environmental policy is implemented, A_d remains constant and A_c grows steadily over time. Final good consumption also grows steadily. It is important to note that while all R&D switches to clean technologies, the relative (and absolute) production of dirty inputs does not immediately decline. In particular, in this case it takes over 70 years for 90% of all energy consumption to be met from clean (nonfossil) sources. As a consequence, and perhaps more importantly, temperature increases do not cease immediately. Figure 1B show that under this optimal profile, temperature increases a further 2.8°C in 85 years, and then decreases slowly thereafter. Figure 1C shows that the optimal input tax schedule increases very slowly at the beginning until it reaches approximately 5.2%

in 85 years; thereafter, it decreases (beyond the horizon covered in the figure, it reaches 0 when the increase in temperature has returned back to 0). Finally, Figure 1D shows that the optimal profit tax decreases rapidly and reaches zero after 26 years; thereafter, equilibrium R&D is directed to clean technologies without further intervention.

An interesting implication of the simple calibration exercise is that the main quantitative conclusions of this model of directed technical change are broadly similar for a range of discount factors the previous literature considered (and found to matter considerably for the policy recommendations with exogenous technology). To illustrate this, we show the results of two alternative calibrations with discount rates of $\rho = 0.001$ (similar to that used in the the Stern report) and $\rho = 0.015$ (similar to that assumed by Nordhaus in the DICE model). Figures 2 and 3 show that the broad qualitative features of optimal policy are across these different values of the discount rate; most notably, in each case, optimal policy involves all R&D resources being immediately devoted to clean technologies (and essentially all production eventually using clean technologies). The results for this range of discount rates are similar when we consider perhaps more realistic and higher elasticities, for example, $\varepsilon = 5$.

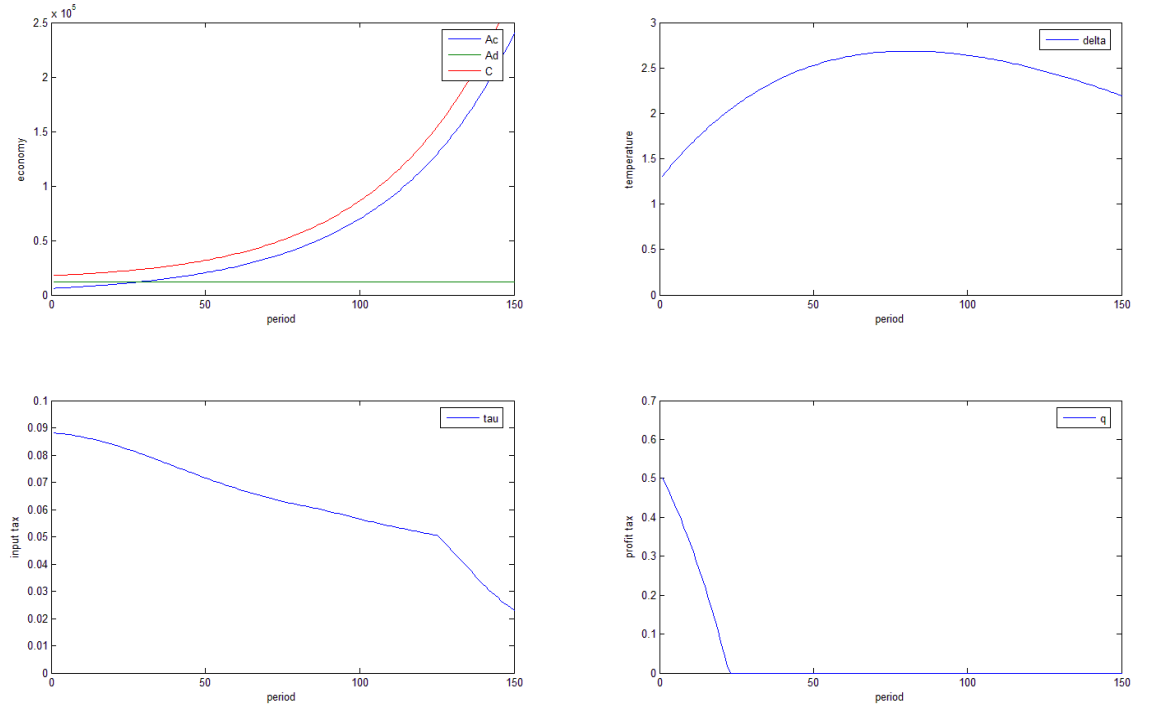


Figure 2: Results from a calibration with $\rho = 0.001$ under the optimal environmental regulation.

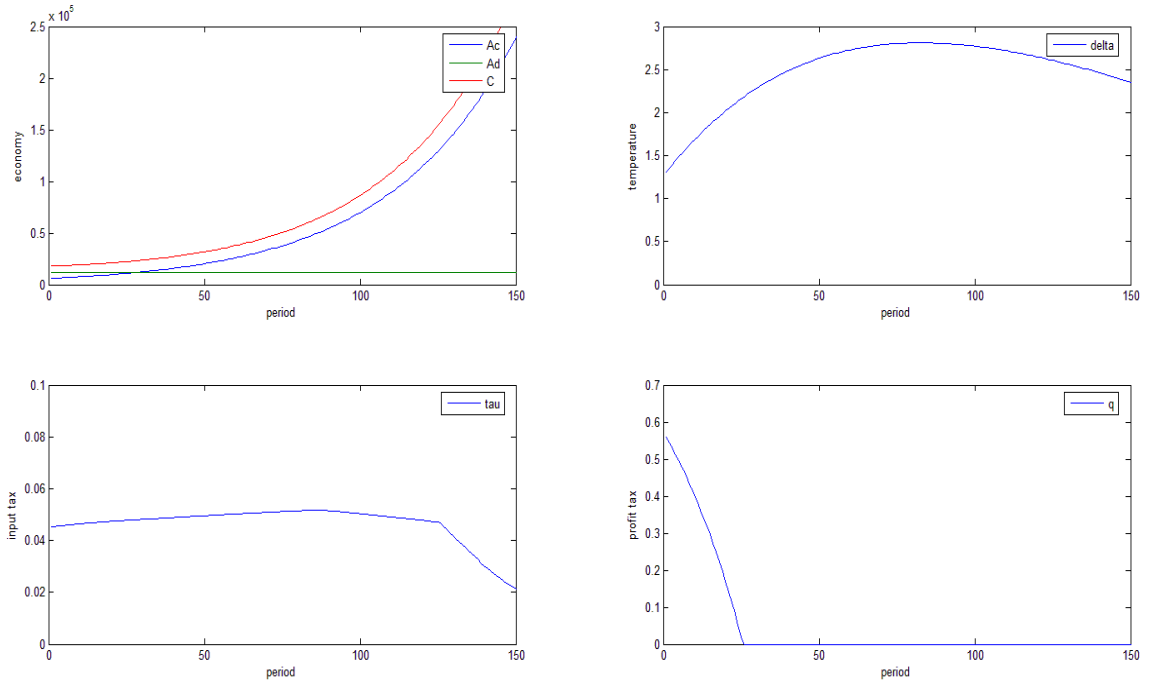


Figure 3: Results from a calibration with $\rho = 0.015$.

Panels A in both Figure 2 and Figure 3 show that under the optimal environmental regulation, R&D again switches to clean technologies immediately. The discount rate naturally affects the level of the input tax; it is lower when ρ is higher, since in this case the planner does not wish to sacrifice as much current growth to increase the quality of the environment (see panels C in Figures 2 and Figure 3). As a consequence, temperature increases slightly more when $\rho = 0.015$ than when $\rho = 0.001$ (panels B in Figures 2 and Figure 3). Nevertheless, the most notable feature here is the similarity between the qualitative and quantitative features of optimal policy across these different discount rates.

Naturally, as the discount rate is raised further, the switch to clean technologies may occur more slowly over time. For example, when $\rho = 0.02$ and $\varepsilon = 3$, the switch of R&D to clean technologies occurs progressively between periods 44 and 53; as a consequence there is more dirty input produced early on. This is partly counteracted by a higher input tax during the intervening period (reaching more than 45%). Notably, however, even when $\rho = 0.02$, the switch to R&D directed towards clean technologies occurs immediately for slightly higher but still reasonable values of ε , for example for $\varepsilon \geq 5$.

2.4.4 Comparison to undirected technical change

We now illustrate how directed technical change affects the optimal policy and the costs of delay in environmental intervention. For comparison, as in subsection 2.3, consider a world without directed technical change, that is, a world where technical change does not respond to economic incentives. Instead, a fixed fraction $s_c = \eta_c / (\eta_c + \eta_d)$ of scientists is always allocated to innovation in clean sector's machines. Then both clean and dirty technologies will always grow at rate $\gamma\tilde{\eta}$. Consequently, in the absence of an input tax, dirty input production will grow over time and lead the economy to a disaster. Thus, as in subsection 2.3, the only way to avoid an environmental disaster is by imposing a permanent tax on the use of the dirty input (a carbon tax). Moreover, because the dirty technology is improving over time, this input tax needs to increase over time. Also note that without directed technical change, the growth rate of the economy will be bounded above by $\gamma\tilde{\eta}$ in the substitutability case, or equal to 0 in the complementarity case. This establishes (proof in the text):

Proposition 9 *With undirected technical change, when $\varepsilon > 1$, optimal environmental regulation involves a growth rate always less than $\gamma\tilde{\eta}$. When $\varepsilon < 1$, optimal environmental regulation involves no growth in the long run. In both cases, the optimal input tax is permanent and increasing over time.*

This proposition therefore further clarifies how directed technical change acts as a force favoring a strong policy intervention in the short run in order to prompt a switch towards innovation directed at clean technologies. This is because the sooner the economy can transition to clean technologies, the more limited is the “transition cost”. Equally important is that when technical change is directed, the optimal policy can keep A_d constant while A_c grows, so the optimal input tax will be decreasing over time. In contrast, with undirected technical change, A_d will keep increasing over time, and therefore the tax must also increase steadily. This proposition therefore shows that the ability of researchers to direct their innovations significantly increases the importance of timely policy intervention and that this conclusion contrasts with those that follow from models with exogenous technology (where the general consensus is that interventions should be gradual and the cost of delay is typically ignored or viewed as second-order).

To further illustrate the implications of directed technical change on the structure of optimal environmental regulation (and the resulting allocations), we now report the results of another calibration exercise, with the only difference from the benchmark being that a constant fraction $\eta_c / (\eta_c + \eta_d)$ of scientists is now allocated to the clean sector regardless of policy (which is an implication of undirected technical change). The results are reported in Figure 4, which, most notably, shows that in contrast to the economy with directed technical change (Figures 1, 2 and 3 above), here the average productivity of clean technologies, A_c , never catches up with

the average productivity of dirty inputs, A_d . As a consequence, the level of production of dirty inputs remains high and the temperature increases to levels very close to those that would create an environmental disaster. Finally, because research cannot be redirected towards clean technologies, optimal policy relies on the input tax to reduce environmental degradation; as a result, the optimal input tax increases over time and becomes much higher than in the case with directed technical change (Figure 1).

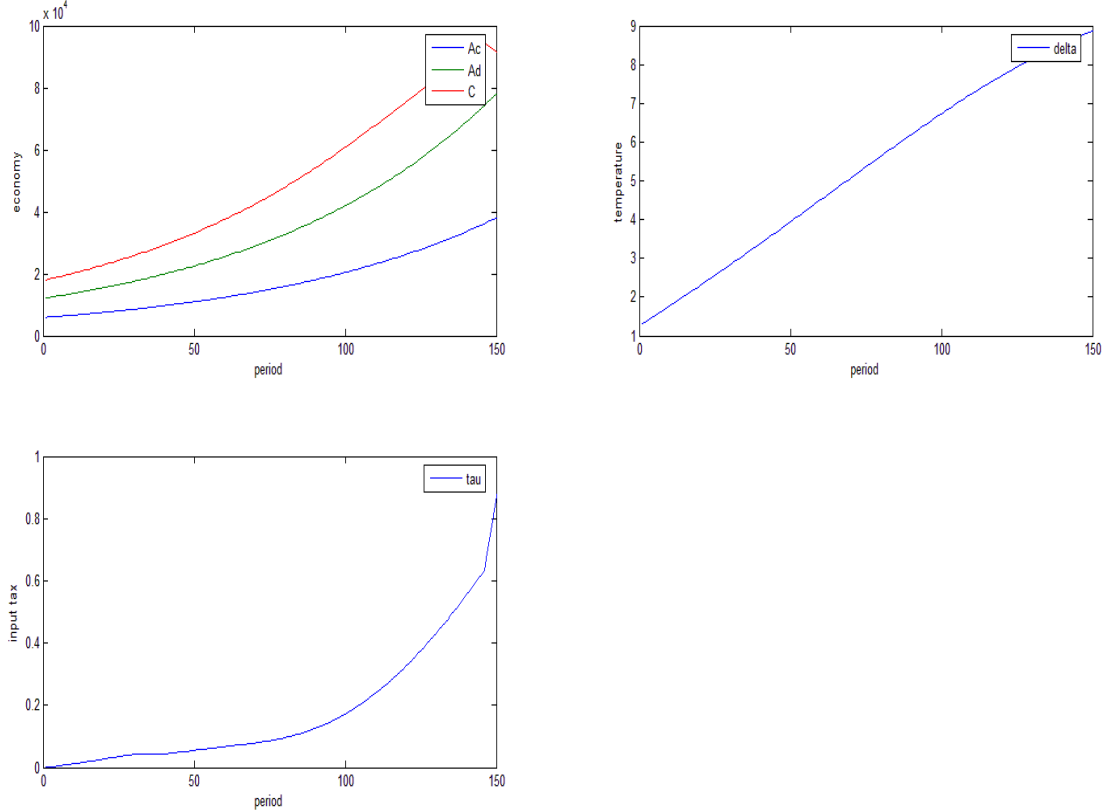


Figure 4: Results for the model with undirected technical change for benchmark parameter values.

Table 1 in subsection 2.3 quantified the costs of delay in terms of the number of additional periods of slow growth that this delay would induce. We can now compute the welfare costs of delaying implementation of the optimal policy. For this purpose, we repeated the same simulation as above, except that the profit and input taxes are assumed to be equal to zero during the first 20 periods (so that the simulations show “optimal policies” after 20 periods delay).³¹

³¹In order to make the comparison more meaningful, we maintain the optimal subsidy on machines, which corrects for the standard monopoly distortions, during the first 20 periods as well.

The results are shown in Figure 5. Figure 5A indicates that A_d increases only during the first twenty periods (before the optimal policy is implemented); thereafter A_c increases when all R&D is optimally directed to the clean sector. Consumption growth is significantly reduced following the introduction of environmental regulation, and is significantly lower because of the delay in intervention. Figure 5B shows that the temperature increases rapidly and reaches a much higher level than the case without delay. Figure 5C shows that delayed intervention also makes the optimal input tax much higher than otherwise, while Figure 5D shows that it further necessitates a longer duration for the profit tax (because during the period of delay, the gap between clean and dirty technologies increases).

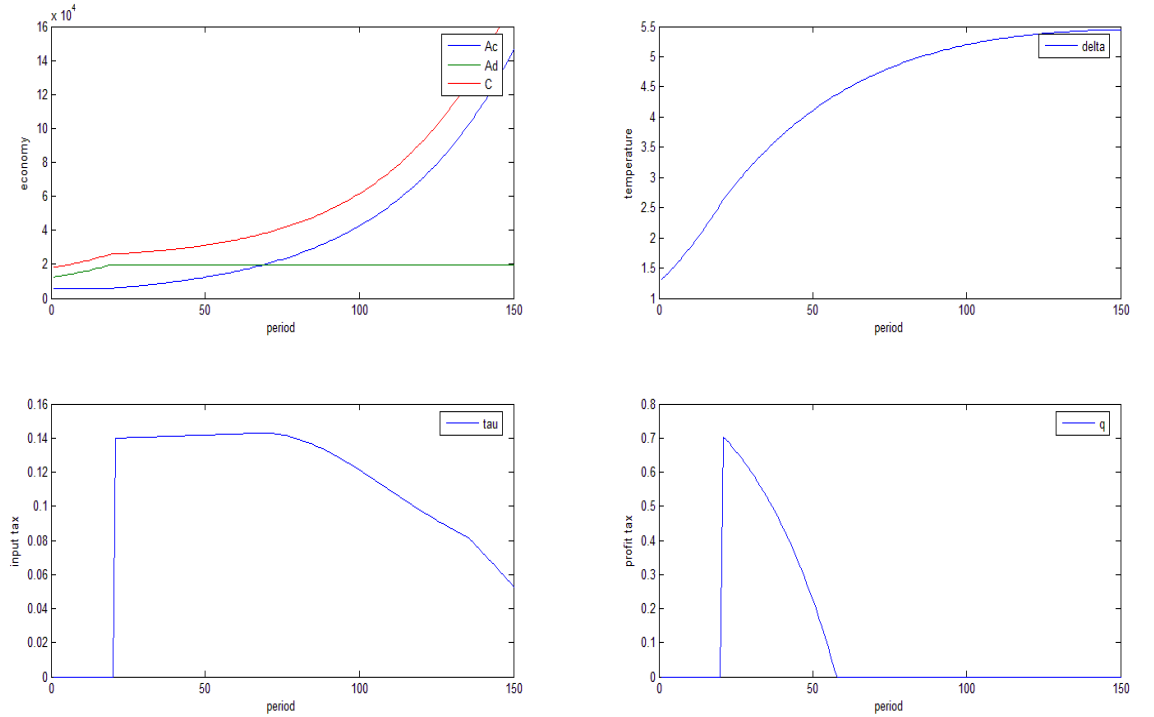


Figure 5: Consequences of delayed intervention (benchmark parameter values).

Table 2 shows the welfare costs of delay in terms of its consumption equivalent, that is, by how much consumption without delay should be reduced per period to give exactly the same level of welfare as the policy that involves delayed intervention (we assume that when intervention starts, it takes the form of optimal environmental regulation as characterized earlier in this subsection). Different numbers in the table correspond to percentage reductions in consumption for different values of the substitutability parameter ε (changing accordingly the initial value A_{ct-1} and A_{dt-1}) and for different numbers of periods of delay. The table clearly

shows that delay costs are substantial, and increase considerably with the duration of the delay and the elasticity of substitution between the two inputs.³² Interestingly, we also find that the welfare costs of delay are largely due to the slow growth that delayed intervention induces (as characterized this subsection 2.3) rather than the further degradation of the environment. This again highlights the importance of directed technical change for the results reported here.

Table 2: Welfare costs of delayed intervention in function of the elasticity of substitution

(Percentage reductions in consumption relative to immediate intervention)

delay\(ε	2	3	5	10
5	1.6	3.0	5.1	7.4
10	2.5	5.3	8.9	12.8
20	4.1	8.5	14.1	19.9
30	5.4	10.7	17.4	24.1

In Table 3 we repeat the same exercise keeping the elasticity of substitution at $\varepsilon = 5$, but varying the discount rate ρ . The cost of delay is substantial even for relatively high discount rates, and naturally, it increases significantly as the discount rate decreases. Note also that the variations in the delay cost are as large as, or greater than, the magnitudes in Table 2; this suggests that the elasticity of substitution between clean and dirty input is as important as the discount rate when assessing the costs of delaying intervention.

Table 3: Welfare costs of delayed intervention in function of the discount rate

(Percentage reductions in consumption relative to immediate intervention)

delay\(ρ	0.001	0.01	0.015
5	5.4	3.0	1.9
10	10.0	5.3	3.1
20	17.6	8.5	4.5
30	23.7	10.7	4.6

Finally, we briefly discuss the welfare costs of relying solely on the input (carbon) tax instead of combining it with the profit tax (which is the pattern of optimal environmental policy in Proposition 8). Table 4 shows that the welfare loss from relying only on the input tax, which in turn leads to excessive reductions in production, is nonnegligible, though small relative to the costs reported in Tables 2 and 3. Interestingly, the welfare loss is higher the

³²When the two inputs are close substitutes, further advances in dirty technologies that occur before the optimal policy is enacted turn out to not contribute much to aggregate output once clean technologies have become sufficiently more advanced than dirty technologies.

smaller the elasticity of substitution, ε , because when the elasticity is higher a relatively small input tax is sufficient to redirect R&D to clean technologies.

Table 4: Welfare costs of relying only on the input tax
(Percentage reductions in consumption relative to optimal policy)

ε	2	3	5	10
cost	0.8	0.6	0.3	0.1

3 Directed technical change with exhaustible resources

Polluting activities often make use of exhaustible resources such as oil or coal. In this section we analyze a variant of our basic model where dirty input production uses an exhaustible resource. Exhaustibility of polluting resources may help prevent an environmental disaster because it increases the cost of using the dirty input even without policy intervention. In particular, we will show that the presence of an exhaustible resource can prevent a disaster in the laissez-faire equilibrium when the two inputs are sufficiently substitutable. However, when the two inputs are complementary, the resource constraint tilts innovation towards the dirty technology, but also prevents long-run sustainable growth.

More formally, we amend our basic model by assuming that the dirty input is now produced according to the technology:

$$Y_{dt} = R_t^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di, \quad (34)$$

where R_t is the flow consumption of the exhaustible resource at time t , and $\alpha_1 + \alpha_2 = \alpha$ (so the labor share in the production of intermediary input remains $1 - \alpha$). The basic model is then simply a subcase of this extended model with $\alpha_2 = 0$. We assume that the exhaustible resource can be directly extracted at a cost $c(Q_t)$ in term of units of final good, where Q_t denotes the resource stock at date t , and c is a decreasing function of Q . This specification makes the exhaustible resource also subject to the “tragedy of the commons”: the price of the exhaustible resource does not reflect its scarcity value. This assumption is adopted to simplify the exposition. When we characterize optimal environmental regulation below, this scarcity value will feature in the social planner’s optimization problem. Given the amount of extraction, the evolution of the exhaustible resource is given by the difference equation:

$$Q_{t+1} = Q_t - R_t \quad (35)$$

In the first subsection we analyze the laissez-faire equilibrium of this augmented model, and in the second subsection we derive the socially optimal environmental regulation.

3.1 The laissez-faire equilibrium

The structure of equilibrium remains mostly unchanged, particularly the equilibrium demands for the two types of inputs, and the production of clean inputs. The equilibrium demands for machines used in the dirty sector become:

$$x_{dit} = \left(\frac{(\alpha_1)^2 p_{dt} R_t^{\alpha_2} L_{dt}^{1-\alpha}}{\psi} \right)^{\frac{1}{1-\alpha_1}} A_{dit}.$$

Profits of monopolists and expected profits from research in the dirty sector are also modified accordingly.

The relative profit abilities of innovation in the clean or in the dirty sector reflect the same three effects as before: the direct productivity effect, the price effect and the market size effect identified above. The only change relative to the baseline model is that the resource stock now affects the magnitude of the price and market size effects. In particular, as the resource stock declines, the effective productivity of the dirty input also declines and its price increases. We show in Appendix F that the price ratio of dirty to clean input is now given by:

$$\frac{p_{ct}}{p_{dt}} = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_{dt}^{1-\alpha_1}}{c(Q_t)^{\alpha_2} \alpha^{2\alpha} A_{ct}^{1-\alpha}}, \quad (36)$$

and the relative employment in the two sectors becomes

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{c(Q_t)^{\alpha_2} \alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}} \right)^{(\varepsilon-1)} \frac{A_{ct}^{-\varphi}}{A_{dt}^{-\varphi_1}} \quad (37)$$

(where $\varphi_1 \equiv (1 - \alpha_1)(1 - \varepsilon)$) so that the share of labor allocated to the dirty sector decreases with the extraction cost only when the two inputs are substitutes. Using these expressions, we obtain the ratio of expected profits from research in the two sectors, which will determine the direction of equilibrium research, as (see Appendix F):

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \kappa \frac{\eta_c c(Q_t)^{\alpha_2(\varepsilon-1)}}{\eta_d} \frac{(1 + \gamma \eta_c s_{ct})^{-\varphi-1}}{(1 + \gamma \eta_d s_{dt})^{-\varphi_1-1}} \frac{A_{ct-1}^{-\varphi}}{A_{dt-1}^{-\varphi_1}} \quad (38)$$

where $\kappa \equiv \frac{(1-\alpha)\alpha}{(1-\alpha_1)\alpha_1^{(1+\alpha_2-\alpha_1)/(1-\alpha_1)}} \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}} \right)^{(\varepsilon-1)}$.

The main difference from the profit ratio in the baseline model is the term $c(Q_t)^{\alpha_2(\varepsilon-1)}$ in the right hand side of (38). This new term, together with the assumption that $c(Q_t)$ is decreasing in Q_t , implies the following proposition (proof in the text):

Proposition 10 *As the resource stock gets depleted over time, innovation incentives in the clean sector increase when two inputs are substitute ($\varepsilon > 1$) but decrease when the two inputs are complements ($\varepsilon < 1$).*

Intuitively, resource depletion increases the relative price of the dirty input, and thus reduces the market for the dirty input. In the substitutability case this encourages innovation in the clean sector. In fact, in the laissez-faire equilibrium, innovation will ultimately occur in the clean sector only (either because the extraction cost increases sufficiently rapidly, inducing all innovation to be directed at clean machines, or because the resource stock gets fully depleted in finite time). In this case, the dirty input is not essential to final production and therefore, provided that initial environmental quality is sufficiently high, an environmental disaster can be avoided while the economy achieves positive long-run growth at the rate $\gamma\eta_c$. In contrast, in the complementarity case the increase in the relative price of the dirty input encourages innovation in the dirty sector. In addition, in this case the dirty input remains essential for final production. Thus positive growth requires an ever increasing rate of extraction, which in turn leads to the exhaustion of the natural resource in finite time. This in turn prevents positive long-run growth. This discussion therefore establishes (proof in the text):

- Proposition 11** 1. *When the two inputs are substitutes ($\varepsilon > 1$), innovation in the long-run will be directed towards the clean sector only and the economy will grow at a rate $\gamma\eta_c$. Provided that \bar{S} is sufficiently high, an environmental disaster is avoided under laissez-faire.*
2. *When the two inputs are complements ($\varepsilon < 1$), economic growth is not sustainable in the long-run.*

Proof. See Appendix G. ■

The most important result in this proposition is that when an exhaustible resource is necessary for production of the dirty input, the market generates incentives for research to be directed towards the clean sector, and these market-generated incentives may be sufficient for the prevention of environmental disaster. This contrasts with the result that an environmental disaster was unavoidable under laissez-faire without the exhaustible resource. Therefore, to the extent that increasing price of oil and oil extraction will create a natural move away from dirty inputs (and other activities creating environmental degradation), the implications of growth are not as damaging to the environment as our baseline model suggests. Nevertheless, because of the environmental and the knowledge externalities, even though an environmental disaster can be averted, the equilibrium is still Pareto suboptimal (even if it avoids an environmental disaster) and the next subsection discusses the structure of optimal environmental regulation with an exhaustible resource.

3.2 Optimal environmental regulation with exhaustible resources

We now briefly discuss the structure of optimal policy with exhaustible resource. The social planner again maximizes (1), now subject to subject to the constraints: (4), (7), (8), (9), (15),

(34) (which replaces (5)), the resource constraint $Q_t \geq 0$,

$$C_t = Y_t - \psi(X_{ct} + X_{dt}) - c(Q_t)R_t, \quad (39)$$

and the law of motion of the resource stock given by (35).

As in subsection 2.4, the social planner will correct for the monopoly distortion by subsidizing the use of machines and will again introduce a wedge between the shadow price of the dirty input and its marginal product in the production of the final good, equivalent to a tax $\tau_t = \omega_{t+1}\xi/\lambda_t p_{dt}$ on dirty input production. In addition, as noted above, we have assumed that the private cost of extraction as given by $c(Q)$ and does not incorporate the scarcity value of the exhaustible resource. The social planner will naturally recognize this scarcity value and will use a “resource tax” to create a wedge between the cost of extraction and the social value of the exhaustible resource.

We denote the Lagrange multiplier for the equation (35) by m_t . Then, the first-order condition with respect to R_t implies:

$$\alpha_2 p_{dt} R^{\alpha_2-1} L_d^{1-\alpha} \int_0^1 A_{di}^{1-\alpha_1} x_{di}^{\alpha_1} di = \frac{m_t}{\lambda_t} + c(Q),$$

where recall that $p_{jt} = \lambda_{jt}/\lambda_t$. Here, the wedge m_t/λ_t is the value, in time t units of final good, of one unit of resource at time t .

The shadow value of one unit of natural resource at time t is then determined by the first-order condition with respect to Q_t , which is

$$m_t = m_{t-1} + \lambda_t c'(Q_t) R_t$$

and thus implies

$$m_t = m_\infty + \sum_{v=t+1}^{\infty} \lambda_v (-c'(Q_v)) R_v.$$

(where m_∞ is the limit of m_t when $t \rightarrow \infty$).

Thus achieving the social optimum requires a resource tax equal to

$$\theta_t = \frac{m_t}{\lambda_t c(Q_t)} = \frac{m_\infty - \sum_{v=t+1}^{\infty} \frac{1}{(1+\rho)^{v-t}} c'(Q_v) R_v \partial u(C_v, S_v) / \partial C}{c(Q_t) \partial u(C_t, S_t) / \partial C}. \quad (40)$$

In particular, the optimal resource tax is always positive. This establishes:

Proposition 12 *The social planner can implement the social optimum through a tax on the use of the dirty input, a tax on profits in the dirty sector, a subsidy on the use of all machines and a resource tax (all proceeds from taxes/subsidies being redistributed/financed lump-sum). The resource tax must be maintained forever.*

Proof. The proof is similar to that of Proposition 7 and is omitted to save space. ■

4 Global environmental externalities and policy coordination

We now study a two-country extension of the baseline model in order to investigate the implications of environmental and knowledge externalities on the need for global policy coordination. We ask whether environmental regulation in one set of countries (the more advanced North) can be sufficient to avoid environmental disasters and how this conclusion is affected by the presence or absence of international trade.

The world economy consists of two “countries,” North and South, and we index all variables (except the quality of the environment, which is common to all countries) with a superscript $k \in \{N, S\}$. We think of the North as the technological leader and of the South as representing countries behind the world technology frontier (that are less productive and less prosperous).

The environmental quality S enters the utility of households in both countries in the same way as in (1). Most importantly, environmental externalities are global, thus the law of motion of the quality of the environment is a function of the total dirty input production in the two countries. In particular, equation (9) is now replaced with

$$S_{t+1} = \max\{0, \min\{-\xi(Y_{dt}^N + Y_{dt}^S) + (1 + \delta)S_t, \bar{S}\}\}. \quad (41)$$

The North is identical to the economy described in the baseline model of Section 2. To simplify the discussion, we assume that the South has exactly the same production structure (the same technology for final production, (4), the same technology for the production of dirty and clean inputs, (5), and the same marginal cost of producing machines, $\psi \equiv \alpha^2$ units of the final good), and also has $s = 1$ scientists, but Southern scientists work only on imitating already developed technologies in the North. This assumption captures the intuitive notion that the South is technologically less advanced and adopts the innovations developed in the North (perhaps with some delay).³³

Southern scientists direct their (imitation) research towards dirty or clean machines in the same way that Northern scientists did. In particular, once they choose a particular sector, as with the Northern scientists, they are randomly allocated to a single machine in the sector of their choice, and in sector $j \in \{c, d\}$ they have a probability $\kappa_j \in (0, 1)$ of successfully imitating this machine (again without congestion, so there is at most one scientist per machine). If they are successful, they will have imitated the frontier machine in the North, thus for machine i in sector $j \in \{c, d\}$, they will have access to the machine of quality A_{jit}^N . Moreover, they will be given a one period monopoly rights over this successfully imitated machine (for use in the South only). For a machine type for which there has not been as successful innovation, monopoly rights are allocated to a Southern entrepreneur drawn at random, and this entrepreneur will

³³Naturally, we could allow scientists in the South to also choose whether to work towards original innovations or to imitate Northern technologies. We do not introduce this choice to simplify the exposition and focus on the effects of Northern technological advances on technology adoption decisions in the South, which is a crucial global interaction that has not been highlighted by previous analyses.

use the technology from the previous period A_{jit-1}^S . Therefore, the structure of technological advances in the South is very similar to that in the North, with the only difference being that “success” brings a machine of quality equal to the frontier quality in the North rather than an incremental improvement over the current machine quality in the South.

Given these assumptions, when s_{jt}^S scientists in the South undertake research in sector $j \in \{c, d\}$ at time t , the law of motion of average technology of sector $j \in \{c, d\}$ in the South evolves according to:

$$A_{jt}^S = \kappa_j s_{jt}^S A_{jt}^N + (1 - \kappa_j s_{jt}^S) A_{jt-1}^S. \quad (42)$$

This equation, together with the law of motion of productivity in the North, (15), gives the evolution of productivity levels in the two sectors in the North and the South.

In the remainder of this section, we first investigate this global economy without trade in inputs, and then turn to the implications of international trade for the environment and the need for global policy coordination.

4.1 Preventing environmental disaster without global policy coordination

Suppose to start with that the North follows an environmental policy (sequences of taxes) denoted by $\{\tau_t^N, q_t^N\}$, where τ_t^N is input tax at time t and q_t^N is the profit tax on dirty innovation (both of those applied only in the North). There is no global policy coordination, so that these policies do not apply to producers in the South. Instead, to capture the relevant situation in which environmental regulations are more lax in developing countries, we assume that the Southern firms operate under *laissez-faire*.

Since in the South all machines are also supplied by monopolists, the static equilibrium in both the South and the North, given technology levels, is the same as that given by our analysis in subsection 2.2. In addition, as with the researchers in the North, the decision of Southern scientists to direct their (imitation) activity towards dirty or clean inputs will be determined by the relative profitability of having access to monopoly rights (for one period) in the two sectors. The expected profits from these monopoly rights in the South are denoted by Π_{jt}^S for sector $j \in \{c, d\}$ at time t and are given by an equation very similar to (16), adapted only to the different innovation possibilities frontier facing Southern scientists:

$$\Pi_{jt}^S = \kappa_j (1 - \alpha) \alpha (p_{jt}^S)^{\frac{1}{1-\alpha}} L_{jt}^S A_{jt}^N.$$

The crucial difference here from (16) is that successful imitation will lead to the imitation of the currently available technology in the North, which explains the term A_{jt}^N at the end. Consequently, the profitability of imitating clean relative to dirty technologies in the South is determined by the ratio

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c (p_{ct}^S)^{\frac{1}{1-\alpha}} L_{ct}^S A_{ct}^N}{\kappa_d (p_{dt}^S)^{\frac{1}{1-\alpha}} L_{dt}^S A_{dt}^N} = \frac{\kappa_c (A_{ct}^S)^{-\varphi-1} A_{ct}^N}{\kappa_d (A_{dt}^S)^{-\varphi-1} A_{dt}^N}. \quad (43)$$

If this ratio is greater than 1, then imitation will be directed to the clean sector only; and if it is smaller than one, imitation will be directed towards the dirty sector only (finally, if it is equal to 1 imitation can occur in both sectors simultaneously).³⁴

Equation (43) shows that the relative profitability of imitation of different types of machines is shaped by the same market size and price effects that determined innovation in the North. However, there is also a different type of *knowledge externality*, reflected by the term (A_{ct}^N/A_{dt}^N) on the right hand side of (43), now resulting from the innovation decisions in the North. Intuitively, profits from imitation are proportional to the target productivity level, which here is the technology in the North, and thus, this knowledge externality favors imitation in the sector that is relatively more advanced in the North. In particular if the quality of clean machines becomes much higher than the quality of dirty machines in the North, this will create an incentive for the South to imitate in the clean sector. This last observation is important for understanding why, under certain circumstances, environmental disaster can be avoided without global policy coordination.

More specifically, a key implication of (43) is that if $\varepsilon > 1$ and the North devotes all its research effort to innovation on clean machines, firms in the South will also eventually switch to clean imitation activities, and A_{ct}^S will grow in the long-run at the same rate as A_{ct}^N , namely at rate $\gamma\eta_c$.³⁵ Now, suppose that indeed the North undertakes an environmental policy that redirects all innovation towards the clean sector, but there is no global policy coordination, so that the South remains under *laissez-faire*. To analyze the conditions under which a disaster can be avoided, we can look at the long-run growth rate of dirty input production in the South. The equilibrium production of dirty inputs in the South, given average productivity levels that firms have access to, will be given by the equivalent of (21) from our analysis in subsection 2.3, and thus:

$$Y_{dt}^S = \frac{(A_{ct}^S)^{\varphi+\alpha} A_{dt}^S}{((A_{ct}^S)^\varphi + (A_{dt}^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}}. \quad (44)$$

This expression highlights that in the long run, output of the dirty input in the South will be approximately equal to $Y_{dt}^S \approx (A_{ct}^S/A_{dt}^S)^{\varphi+\alpha}$, which decreases if $\varphi + \alpha < 0$ (that is if

³⁴It may also be useful to write this expression in terms in term of time- $t-1$ productivity levels, which is

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c ((1 - \kappa_c s_{ct}^S) A_{ct-1}^S + \kappa_c s_{ct}^S A_{ct}^N)^{-\varphi-1} A_{ct}^N}{\kappa_d ((1 - \kappa_d s_{dt}^S) A_{dt-1}^S + \kappa_d s_{dt}^S A_{dt}^N)^{-\varphi-1} A_{dt}^N}.$$

³⁵Too see this, note that: (i) A_{ct}^S cannot grow faster than A_{ct}^N since the South cannot do better than imitating the North; (ii) in the long run, A_{ct}^S will in fact grow at the same rate as A_{ct}^N ; to obtain a contradiction suppose it grew more slowly; then (43), together with the fact that A_{dt}^N and therefore A_{dt}^S remain fixed once the North innovates in the clean sector only, would imply that Π_{ct}^S/Π_{dt}^S goes to infinity as $t \rightarrow \infty$; but then imitation in the South would end up occurring in the clean sector only in finite time; this in turn implies that A_{ct}^S and A_{ct}^N must grow at the same rate in the long-run, yielding a contradiction; (iii) the fact that A_{ct}^S and A_{ct}^N grow at the same rate in the long-run in turn implies that Π_{ct}^S/Π_{dt}^S must exceed 1, and consequently, in finite time all imitation in the South will switch to the clean sector.

$\varepsilon > 1/(1 - \alpha)$), and otherwise grows at rate $(1 + \gamma\eta_c)^{\varphi+\alpha} - 1$.

Given this observation, the main insights here parallel those in subsection 2.3. In particular, as in subsection 2.3, when $\varepsilon < 1/(1 - \alpha)$, the global production of dirty input will grow to infinity (since production over the dirty input in the South grows steadily over time). Consequently, an environmental disaster is unavoidable. In contrast, when $\varepsilon > 1/(1 - \alpha)$, i.e., when the two inputs are strong substitutes, environmental disaster can be avoided if \bar{S} is sufficiently large. As all innovation in the North is directed to clean inputs, in this case, the production of dirty inputs in the North stops growing. The analysis in this subsection shows that Southern scientists will ultimately imitate clean technologies in the North. Moreover, equation (44) then implies that the production of dirty inputs in the South will also stop growing. Thus with a sufficiently high level of initial environmental quality, a global environmental disaster can be prevented even without global policy coordination. The role of directed technical change in this result is clear, since it is directed technical change that allows the North to redirect innovation towards clean technologies, and it is also the ability of Southern scientists to redirect their imitation activity towards clean technologies that enables Southern firms to switch to clean frontier technologies once these have become sufficiently advanced.

We summarize this result in the next proposition (proof omitted):

Proposition 13 *In the two-country case when $\varepsilon > 1/(1 - \alpha)$, a policy $\{\tau_t^N, q_t^N\}$ in the North that would direct innovation towards clean technologies only, is sufficient to avoid a disaster without taxation in the South provided that \bar{S} is sufficiently high. If $1 < \varepsilon < 1/(1 - \alpha)$ such a policy cannot prevent a disaster.*

Proposition 13 shows that a global environmental disaster can be avoided using simple policies (without global policy coordination). Clearly, optimal environmental regulation will be more complex in this case and will involve global policy coordination. For completeness, we characterize the structure of optimal environmental regulation in Appendix I (from the point of view of a global social planner interested in maximizing the sum of the utilities of households in both countries given by (1)). The following proposition summarizes the main results:

Proposition 14 *The global social optimum can be implemented through a combination of profits and input taxes both in the North and in the South, and a subsidy to machine consumers (against the standard monopoly distortion). If $\varepsilon > 1/(1 - \alpha)$, all optimal environmental regulation (taxes) are temporary.*

4.2 International trade and pollution havens

The argument that knowledge spillovers should induce the South to follow the North in its switch to clean technologies may be counteracted by international trade, creating a greater need for global policy coordination in environmental policies. In particular, free international

trade between the North and the South, combined with environmental regulation in the North, creates a comparative advantage in dirty input production in the South. Loosely speaking, in this case, the South may become a “pollution haven”. This in turn may make an environmental disaster more likely.

To investigate these issues, we now allow international trade in inputs between the North and the South. Our model will behave as a Ricardian model within each period, with the productivity of a sector in each country depending on technology and policies. We focus on the case where $\varepsilon > 1$. As in subsection 4.1, we assume (i) that the North follows an environmental policy $\{\tau_t^N, q_t^N\}$ where τ_t^N is a tax on the production of the dirty input in the North, and q_t^N is a tax on profits in the dirty sector in the North, such that the government in the North redirects innovation towards the clean technology only; (ii) that the South remains under pure laissez-faire. But now there is free trade in inputs.

Similarly to (18) in the one country case, equilibrium input production levels are given by:

$$Y_{jt}^k = \left(p_{jt}^k\right)^{\frac{\alpha}{1-\alpha}} A_{jt}^k L_{jt}^k, \quad (45)$$

where $j \in \{c, d\}$, $k \in \{N, S\}$, and p_j^k is the pre-tax price of input j in country k .

The ratio of marginal products of labor in sectors c and d in country k is then equal to.

$$\frac{MPL_c^k}{MPL_d^k} = \left(\frac{p_c^k}{p_d^k}\right)^{\frac{1}{1-\alpha}} \frac{A_c^k}{A_d^k}. \quad (46)$$

Whichever country has a higher ratio in (46) will have a comparative advantage in the clean sector; the other country will have a comparative advantage in the dirty sector.

Next note that free trade also implies that the post-tax price for each input ($j = c, d$) must be equalized in the North and the South, so that:

$$p_{ct}^N = p_{ct}^S \text{ and } (1 + \tau_t^N)p_{dt}^N = p_{dt}^S. \quad (47)$$

Thus the South will have a comparative advantage in producing the dirty input at time t if

$$(1 + \tau_t^N)^{\frac{1}{1-\alpha}} \frac{A_{ct}^N}{A_{dt}^N} > \frac{A_{ct}^S}{A_{dt}^S}. \quad (48)$$

This expression implies that both a higher input tax rate τ_t^N in the North and a higher relative quality of clean machines in the North reinforce the South’s comparative advantage in dirty input production. This is the “pollution haven hypothesis” in a world with endogenous technology. In this case, there is a possibility that environmental policy in the North may induce dirty activities to move to the South, which will then export part of its dirty input production back to the North. If innovation and imitation decisions also reinforce this pattern, this type of international trade may increase the likelihood of an environmental disaster.

To illustrate the main ideas, let us first focus on the case in which both countries fully specialize, that is, the North only produces clean inputs and the South only produces dirty inputs (the more general case is studied in Appendix H, which also provides a sufficient condition for complete specialization to occur in equilibrium). Under complete specialization, equilibrium input production levels in the South and the North are given by:

$$Y_d^S = (p_d^S)^{\frac{\alpha}{1-\alpha}} A_d^S L^S, Y_d^N = 0, Y_c^N = (p_c)^{\frac{\alpha}{1-\alpha}} A_c^N L^N \text{ and } Y_c^S = 0. \quad (49)$$

Therefore, without any environmental regulation in the South and with complete specialization, Southern scientists will target machines in the dirty sector. Then whether the global economy can avoid the disaster will depend on the rate of growth of the production of dirty inputs in the South. Recall that A_c^N grows in the long-run at rate $\gamma\eta_c$, while A_d^N is constant. Since the South imitates the North, this implies that A_d^S will also be constant. Using these observations, we show in Appendix H that in the long-run:

$$Y_d^S \simeq \left(\frac{\alpha^2}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (A_c^N L^N)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} (A_d^S L^S)^{\frac{\varepsilon(1-\alpha)}{\varepsilon(1-\alpha)+\alpha}},$$

which increases at a rate $(1 + \gamma\eta_c)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} - 1 > 0$, implying that an environmental disaster cannot be avoided without global policy coordination. This argument then suggests that an environmental disaster is more likely to occur under free trade than under no-trade (without global policy coordination or environmental regulations in the South). The intuition for this result is essentially given by the “pollution haven” hypothesis: the production of dirty inputs migrates to the South and does not decline despite environmental regulations and innovation in clean technologies in the North. Nevertheless, this result only holds in this sharp form when there is complete specialization. The next proposition shows that, under certain conditions, there exists a complete specialization equilibrium (though this is not the unique equilibrium). We then discuss how “pollution haven”-type ideas apply more generally in Proposition 16.

Proposition 15 *Consider the two-country case with free trade in inputs ($j = c, d$). When the two inputs are strong substitutes ($\varepsilon > 1/(1 - \alpha)$) and A_{c0}^S is sufficiently small, there exists an equilibrium in which any environmental policy $\{\tau_t^N, q_t^N\}$ in the North redirecting technical change to the clean sector is insufficient to avoid an environmental disaster under free trade (though it would have avoided a disaster under autarky).*

Proof. See Appendix H. ■

Proposition 15 shows that there exists an equilibrium in which the South will completely specialize in dirty inputs and in this equilibrium, environmental regulation in the North is not sufficient to prevent an environmental disaster. However, the analysis in Appendix H shows that for the same parameter values, there may also exist another equilibrium in which Southern

scientists coordinate and switch to clean technologies. We next present a complementary result showing that under free trade there is higher production of dirty input in the South even if Southern scientists could fully coordinate to make the switch to clean technologies whenever it is jointly beneficial for them. This result therefore illustrates another, perhaps more robust, facet of the “pollution haven” hypothesis.

Before stating this result, we briefly provide the general intuition. For given technologies, trade opening always induces an increase in the production of the dirty input in the South. The countervailing effect is that it also reduces the production of dirty input in the North (both relative to autarky). Let us focus on the case where the two inputs are highly substitutable, in particular, $\varepsilon > (2 - \alpha) / (1 - \alpha)$ (which implies $\varepsilon > 1 / (1 - \alpha)$). In this case, the global production of the dirty input increases under the following scenarios (at time t): (i) if there is complete specialization in the South and the production of dirty input in the North under autarky was already low to start with,³⁶ or (ii) if there is incomplete specialization in the South, but the South has a “technological” comparative advantage in the production of the dirty input, that is, if $(A_{ct}^N / A_{dt}^N) > (A_{ct}^S / A_{dt}^S)$. If so, then trade opening increases the global production of the dirty input. Whether either of these scenarios apply depends on how technologies in the South will evolve under free trade. Here, we can also show that trade liberalization induces increased imitation in the dirty sector in the South because of the market size effect (the market size for dirty inputs in the South has increased due to trade opening).³⁷ Consequently, in this case, trade opening will necessarily increase the global production of the dirty input both upon impact and subsequently, and thus, it will create greater environmental degradation. Moreover, this conclusion now holds when scientists in the South can coordinate to switch to clean technologies whenever doing so is an equilibrium. This result is stated in the next proposition, which imposes sufficient conditions for both the impact and the subsequent effects to work in the same direction (and increase the global production of the dirty input).

Proposition 16 *Suppose that (a) the two inputs are highly substitutable ($\varepsilon > (2 - \alpha) / (1 - \alpha)$); (b) the amount of dirty input produced in the North under autarky is sufficiently low; (c) the South has an initial technological comparative advantage in dirty inputs, i.e., $A_{c0}^N / A_{d0}^N > (1 + \gamma\eta_c / \kappa_c) / (1 + \gamma\eta_c) > A_{c0}^S / A_{d0}^S$; and (d) whenever there are multiple equilibria with imitation of both clean and dirty technologies, scientists in the South coordinate on imitating clean technologies. Then the level of production of dirty inputs under free trade is always greater than under autarky, and for sufficiently high levels of initial environmental quality, there exist policies $\{\tau_t^N, q_t^N\}$ that direct innovation towards clean technologies in the North can avoid an*

³⁶This would be the case, for example, if the clean technology in the North is already sufficiently advanced at the time of trade opening.

³⁷Here the assumption that $\varepsilon > (2 - \alpha) / (1 - \alpha)$ ensures that the two inputs are sufficiently substitutable to make Southern scientists prefer, all else equal, to innovate/imitate in the sector that is already more advanced in the South.

environmental disaster under autarky but will fail to do so under free trade.

Proof. See Appendix H. ■

5 Conclusion

In this paper we introduced endogenous directed technical change in a growth model with environmental constraints and limited resources. We characterized the structure of equilibria and the dynamic tax policies that achieve sustainable growth or maximize intertemporal welfare. Both the long-run properties of the equilibrium and optimal policies (or the necessary policies to avoid environmental disasters) are related to the degree of substitutability or complementarity between clean and dirty inputs, to whether dirty input production uses exhaustible resources, to initial environmental and resource stocks, and to cross-country technological spillovers.

Our analysis implies that factoring in directed technical change: (i) increases the cost of delaying intervention, particularly in the substitutability case; (ii) calls for the use of profit taxes or other instruments to direct innovation, in addition to the input tax emphasized so far by the literature. Moreover we showed that: (i) in the case where the clean and dirty inputs are substitutes, one can achieve sustainable long-run growth with temporary taxation of dirty innovation and production; (ii) the sooner and stronger the policy response, the shorter the slow growth transition phase; (iii) the use of an exhaustible resource in dirty goods production helps the switch to clean innovation under laissez-faire when the two inputs are substitute, but the opposite holds when the two inputs are complements. A simple calibration of our model suggests that, provided that elasticity of substitution between clean and dirty inputs is sufficiently high, optimal environmental regulation should involve an immediate switch of R&D resources to clean technology, followed by essentially all production switching to clean inputs. This conclusion appears robust to the range of discount rates used in the Stern report and in Nordhaus's work (which lead to very different policy conclusions in models with exogenous technology).

We also used our basic framework to investigate issues of global policy coordination. In a two-country extension where: (a) the two inputs are substitutable in both countries; (b) dirty input production in both countries depletes the global environmental stock; (c) the South imitates technologies invented in the North, then taxing dirty innovation and production in the North only, may be sufficient to avoid a global disaster; however, this is less likely to be true if free trade is allowed between North and South, since in that case taxing the North only may induce full specialization by the South in dirty input production.

Our paper is a first step towards a comprehensive framework that can be used for theoretical and quantitative analysis of environmental regulation with endogenous technology. Several directions of future research appear fruitful. First, it would be useful to develop a more detailed

multi-country model with endogenous technology and environmental constraints, which can be used to discuss issues of global policy coordination and the degree to which international trade should be linked to environmental policies. Second, an interesting direction is to incorporate “environmental risk” into this framework, for example, because of the ex ante uncertainty on the regeneration rate, δ , or on the initial environmental quality, S_0 . Another line of important future research would be to exploit macroeconomic and microeconomic (firm- and industry-level) data to estimate the relevant elasticity of substitution between clean and dirty inputs.

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Appendix A: Equilibrium allocations of scientists

In this Appendix, we characterize the equilibrium allocation(s) of innovation effort across the two sectors and provide a proof of Lemma 1. To illustrate the role of Assumption 1 more clearly, we relax this assumption for now (this assumption is imposed throughout the text). Recall that at time t , the ratio of expected profits from undertaking research in clean technologies over the expected profits from undertaking research in dirty technologies is given by (20) in the text. The characterization and the proof of Lemma 1 will follow by considering three cases.

1. Innovation can occur in the clean sector only (i.e., it could be an equilibrium for innovation to be only in the clean sector, which equivalently implies $s_{dt} = 1$ and $s_{ct} = 0$). This applies when

$$\frac{\eta_c}{\eta_d} (1 + \gamma\eta_c)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} \geq 1,$$

which follows from (20) by using the fact that $s_{dt} = 1$ and $s_{ct} = 0$. This condition is also equivalent to

$$\begin{aligned} \frac{A_{ct-1}}{A_{dt-1}} &\geq \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} (1 + \gamma\eta_c)^{\frac{-(\varphi+1)}{\varphi}} \quad \text{when } \varepsilon > 1 \\ \frac{A_{ct-1}}{A_{dt-1}} &\leq \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} (1 + \gamma\eta_c)^{\frac{-(\varphi+1)}{\varphi}} \quad \text{when } \varepsilon < 1. \end{aligned}$$

2. Innovation can occur in the dirty sector only. With the same reasoning as in the previous case, this applies when

$$\frac{\eta_c}{\eta_d} \left(\frac{1}{1 + \gamma\eta_d s_{dt}} \right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} \leq 1$$

or equivalently, when:

$$\begin{aligned} \frac{A_{ct-1}}{A_{dt-1}} &\leq \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \quad \text{when } \varepsilon > 1 \\ \frac{A_{ct-1}}{A_{dt-1}} &\geq \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \quad \text{when } \varepsilon < 1. \end{aligned}$$

3. Finally innovation can occur in both sectors if

$$\frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} \right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} = 1.$$

If $\varepsilon = \frac{2-\alpha}{1-\alpha}$ (that is, if $\varphi + 1 = 0$), this equality holds if and only if $\frac{\eta_c}{\eta_d} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi} = 1$, in which case any s_{ct} is an equilibrium allocation of scientists.

If $\varepsilon \neq \frac{2-\alpha}{1-\alpha}$, the only possible candidate equilibrium with scientists directing research towards both sectors must have the share of scientists working in the clean sector as:

$$s_{ct} = \frac{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} (1 + \gamma\eta_d) - 1}{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} \gamma\eta_d + \gamma\eta_c}.$$

If this value is strictly between 0 and 1, then the candidate equilibrium would be an actual equilibrium. This in turn requires:

$$\begin{aligned} (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} &< \frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} \quad \text{when } \varepsilon < 1 \text{ or } \varepsilon > \frac{2-\alpha}{1-\alpha} \\ (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} &< \frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} \quad \text{when } 1 < \varepsilon < \frac{2-\alpha}{1-\alpha}. \end{aligned}$$

This leads to following characterization of equilibrium innovation allocations between the two sectors:

- Suppose $\varepsilon > \frac{2-\alpha}{1-\alpha}$. If $\frac{A_{ct-1}}{A_{dt-1}} > (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the clean sector. If $\frac{A_{ct-1}}{A_{dt-1}} = (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$ or if $\frac{A_{ct-1}}{A_{dt-1}} = (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation occurs only in the clean sector or only in the dirty sector. If $(1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} < \frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation occurs only in the clean sector or only in the dirty sector or in both sectors with

$$s_{ct} = \frac{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} (1 + \gamma\eta_d) - 1}{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} \gamma\eta_d + \gamma\eta_c}.$$

Finally, if $\frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the dirty sector.

- Suppose $\varepsilon = \frac{2-\alpha}{1-\alpha}$. If $\frac{A_{ct-1}}{A_{dt-1}} > \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the clean sector. If $\frac{A_{ct-1}}{A_{dt-1}} = \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation can occur with any s_{ct} . Finally, if $\frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the dirty sector.
- Suppose $1 < \varepsilon < \frac{2-\alpha}{1-\alpha}$. If $\frac{A_{ct-1}}{A_{dt-1}} \geq (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the clean sector. If $(1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} < \frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$,

then innovation occurs in both sectors with

$$s_{ct} = \frac{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} (1 + \gamma\eta_d) - 1}{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} \gamma\eta_d + \gamma\eta_c}.$$

Finally, if $\frac{A_{ct-1}}{A_{dt-1}} \leq (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the dirty sector.

- Suppose $\varepsilon < 1$. If $\frac{A_{ct-1}}{A_{dt-1}} \geq (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, then innovation at time t must occur only in the dirty sector. If $(1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} < \frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$, innovation must occur in both sectors $s_{ct} = \frac{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} (1 + \gamma\eta_d) - 1}{\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi+1}} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\varphi}{\varphi+1}} \gamma\eta_d + \gamma\eta_c}$; and if $\frac{A_{ct-1}}{A_{dt-1}} \leq (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$ innovation at time t must occur only in the clean sector.

This characterization also shows that when $\varepsilon > \frac{2-\alpha}{1-\alpha}$, there may be multiple equilibria but when $\varepsilon < \frac{2-\alpha}{1-\alpha}$, the equilibrium is always uniquely defined. Nevertheless, under Assumption 1, even when $\varepsilon > \frac{2-\alpha}{1-\alpha}$, the equilibrium is unique, since $\frac{A_{ct-1}}{A_{dt-1}} < (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}$. Lemma 1 then follows directly from this characterization and Assumption 1. ■

Appendix B: Proof of Proposition 1

Let us consider the cases where the two inputs are gross substitutes ($\varepsilon > 1$) and complements ($\varepsilon < 1$) separately.

Case $\varepsilon > 1$: Assumption 1 together with the characterization of equilibrium allocation of scientists in Appendix A implies that initially innovation will occur in the dirty sector only ($s_{dt} = 1$ and $s_{ct} = 0$). From (15), this widens the gap between clean and dirty technologies and ensures that $s_{dt+1} = 1$ and $s_{ct+1} = 0$, and so on in subsequent periods. This shows that under Assumption 1, the equilibrium is uniquely defined under laissez-faire and involves $s_{dt} = 1$ and $s_{ct} = 0$ for all t .

Case $\varepsilon < 1$: In this case the result follows from the following lemma:

Lemma 2 *When $\varepsilon < 1$, long-run equilibrium innovation will be in both sectors so that the equilibrium share of scientists in the clean sector converges to $s_c = \frac{\eta_d}{\eta_c + \eta_d}$.*

Proof. Suppose that at time t innovation occurred in both sectors so that $\frac{\Pi_{ct}}{\Pi_{dt}} = 1$. Then

$$\begin{aligned} \frac{\Pi_{ct+1}}{\Pi_{dt+1}} &= \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s_{ct+1}}{1 + \gamma\eta_d s_{dt+1}} \right)^{-\varphi-1} \left(\frac{A_{ct}}{A_{dt}} \right)^{-\varphi} \\ &= \left(\frac{1 + \gamma\eta_c s_{ct+1}}{1 + \gamma\eta_d s_{dt+1}} \right)^{-\varphi-1} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} \right) \end{aligned}$$

Innovation will then occur in both sectors at time $t + 1$ whenever the equilibrium allocation of scientists (s_{ct+1}, s_{dt+1}) at time $t + 1$ is such that

$$\frac{1 + \gamma\eta_c s_{ct+1}}{1 + \gamma\eta_d s_{dt+1}} = \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} \right)^{\frac{1}{\varphi+1}}. \quad (50)$$

This equation defines $s_{ct+1} = 1 - s_{dt+1}$ as a function of $s_{ct} = 1 - s_{dt}$. We next **claim** that this equation has an interior solution $s_{ct+1} \in (0, 1)$ when $s_{ct} \in (0, 1)$ (i.e., when s_{ct+1} is itself interior). First, note that when $\varphi > 0$ (that is, $\varepsilon < 1$), the function $z(x) = x^{\frac{1}{\varphi+1}} - x$ is strictly decreasing for $x < 1$ and strictly increasing for $x > 1$. Therefore, $x = 1$ is the unique positive solution to $z(x) = 0$. Second, note also that the function

$$X(s_{ct}) = \frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} = \frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d (1 - s_{ct})},$$

is a one-to-one mapping from $(0, 1)$ onto $(\frac{1}{1+\gamma\eta_d}, 1 + \gamma\eta_c)$. Finally, it can be verified that whenever $X \in (\frac{1}{1+\gamma\eta_d}, 1 + \gamma\eta_c)$, we also have $X^{\frac{1}{\varphi+1}} \in (\frac{1}{1+\gamma\eta_d}, 1 + \gamma\eta_c)$. This, together with (50), implies that if $s_{ct} \in (0, 1)$, then $s_{ct+1} = X^{-1}(X(s_{ct})^{\frac{1}{\varphi+1}}) \in (0, 1)$, proving the claim at the beginning of this paragraph.

From Appendix A we know that when $\varphi > 0$, the equilibrium allocation of scientists is unique at each t . Thus as $t \rightarrow \infty$, this allocation must converge to the unique fixed point of the function $Z(s) = X^{-1} \circ (X(s))^{\frac{1}{\varphi+1}}$, which is

$$s_c = \frac{\eta_d}{\eta_c + \eta_d}.$$

This complete the proof of the lemma. ■

Now given the characterization of the equilibrium allocations of scientists in Appendix A, under Assumption 1 the equilibrium involves $s_{dt} = 0$ and $s_{ct} = 1$, i.e., innovation occurs initially in the clean sector only. From (15), A_{ct}/A_{dt} will grow at a rate $\gamma\eta_c$, and in finite time, it will exceed the threshold $(1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}}$. Lemma 2 implies that when this ratio is in the interval $\left((1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}}, \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}} \right)$, equilibrium innovation occurs in both sectors, i.e., $s_{dt} > 0$ and $s_{ct} > 0$. Therefore, from the time the critical threshold is exceeded, innovation will occur in both sectors and the share of scientists devoted to the clean sector converges to $\frac{\eta_d}{\eta_d + \eta_c}$. This completes the proof of Proposition 1. ■

Appendix C: Proof of Corollary 1

From the expressions in (21), dirty input production is given by:

$$Y_{dt} = (A_{ct}^\varphi + A_{dt}^\varphi)^{-\frac{\alpha+\varphi}{\varphi}} A_{ct}^{\alpha+\varphi} A_{dt} = \frac{A_{dt}}{\left(1 + \left(\frac{A_{dt}}{A_{ct}} \right)^\varphi \right)^{\frac{\alpha+\varphi}{\varphi}}}.$$

When the two inputs are gross substitutes ($\varepsilon < 1$), we have $\varphi = \varphi^{su} < 0$, whereas when they are complements ($\varepsilon > 1$), we have $\varphi = \varphi^{co} > 0$. Since all innovations occur in the dirty

sector in the substitutability case, but not in the complementarity case, if we start with the same levels of technologies in both cases, at any time $t > 0$ we have $A_{dt}^{su} > A_{dt}^{co}$ and $A_{ct}^{su} < A_{ct}^{co}$, where A_{kt}^{su} and A_{kt}^{co} denote the average productivities in sector k at time t respectively in the substitutability and in the complementarity case, starting from the same initial productivities $A_{k0}^{su} = A_{k0}^{co}$.

Under Assumption 1, we have

$$\left(\frac{A_{dt}^{su}}{A_{ct}^{su}} \right)^{\varphi^{su}} < \frac{\eta_d}{\eta_c} \leq \left(\frac{A_{dt}^{co}}{A_{ct}^{co}} \right)^{\varphi^{co}}$$

so that

$$\begin{aligned} Y_{dt}^{su} &= \frac{A_{dt}^{su}}{\left(1 + \left(\frac{A_{dt}^{su}}{A_{ct}^{su}} \right)^{\varphi^{su}} \right)^{\frac{\alpha}{\varphi^{su}} + 1}} \\ &> \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}} \right)^{\varphi^{co}} \right)} \left(1 + \left(\frac{A_{dt}^{su}}{A_{ct}^{su}} \right)^{\varphi^{su}} \right)^{-\frac{\alpha}{\varphi^{su}}} \\ &> \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}} \right)^{\varphi^{co}} \right)} \\ &> \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}} \right)^{\varphi^{co}} \right)^{\frac{\alpha}{\varphi^{co}} + 1}} \\ &> Y_{dt}^{co}. \end{aligned}$$

Repeating the same argument for $t + 1$, $t + 2, \dots$, we have that $Y_{dt}^{su} > Y_{dt}^{co}$ for all t . This establishes that, under Assumption 1, there will be a greater amount of dirty input production for each t when $\varepsilon > 1$ than when $\varepsilon < 1$, implying that an environmental disaster will occur sooner when the two sectors are gross substitutes, as claimed in the corollary. ■

Appendix D: Proof of Proposition 7

We first derive the expression for τ_t as a function of A_{ct} , A_{dt} , ω_{t+1} and λ_t . Using (27), we have

$$p_{ct}^{1-\varepsilon} + (p_{dt} (1 + \tau_t))^{1-\varepsilon} = 1 \quad (51)$$

(which is just stating that p_{ct} and p_{dt} are the prices of the two inputs relative to the final good, which is chosen as the numeraire). Since, as in the laissez-faire equilibrium, the value of marginal product of labor must be equated in the two sectors, we have

$$p_{ct}^{\frac{1}{1-\alpha}} A_{ct} = p_{dt}^{\frac{1}{1-\alpha}} A_{dt}. \quad (52)$$

Next, (51) combined with (52) leads to the following expression for the equilibrium (pre-tax) price of the dirty input

$$p_{dt} = \frac{A_{ct}^{1-\alpha}}{\left(A_{ct}^\varphi (1 + \tau_t)^{1-\varepsilon} + A_{dt}^\varphi\right)^{\frac{1}{1-\varepsilon}}}, \quad (53)$$

which is decreasing in the tax rate τ_t (whereas the post-tax price is increasing in τ_t). Similarly, the price of the clean input is given by

$$p_{ct} = \frac{A_{dt}^{1-\alpha}}{\left(A_{ct}^\varphi (1 + \tau_t)^{1-\varepsilon} + A_{dt}^\varphi\right)^{\frac{1}{1-\varepsilon}}}, \quad (54)$$

This tax rate is then uniquely given by:

$$\tau_t^{1-\varepsilon} = \left(\frac{\omega_{t+1}\xi}{\lambda_t}\right)^{1-\varepsilon} \left(1 + \left(\frac{A_{dt}^{1-\alpha}}{(1 + \tau_t) A_{ct}^{1-\alpha}}\right)^{1-\varepsilon}\right).$$

In the text we have already shown how the monopoly distortion can be corrected for using a subsidy on the use of machines. To complete the proof of Proposition 7, we simply need to show that, given this subsidy and the input tax, the appropriate profit tax can implement the optimal allocation of scientists. Using (25) in (27), combined with (52), we find that

$$\frac{L_{ct}}{L_{dt}} = (1 + \tau_t)^\varepsilon \left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}, \quad (55)$$

which implies that a higher tax induces greater relative employment in the clean sector.

Next, incorporating the subsidy to the use of machines, we have

$$x_{jit} = \left(\frac{\alpha}{\psi} p_{jt}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt}, \quad (56)$$

so that pre-tax profits are

$$\pi_{jit} = (1 - \alpha) \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} A_{jit} L_{jt}.$$

Then, under a profit tax of q_t on profits in sector d , the ratio of expected profits from innovation in sectors c and d , the equivalent of (20) becomes

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}}\right)^{-\varphi-1} \frac{(1 + \tau_t)^\varepsilon}{(1 - q_t)} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}.$$

An identical argument to that in Lemma 1 implies that innovation at time t occurs in the clean sector only when $\frac{\Pi_{ct}}{\Pi_{dt}} > 1$, in the dirty sector only when $\frac{\Pi_{ct}}{\Pi_{dt}} < 1$, and it may occur in both when $\frac{\Pi_{ct}}{\Pi_{dt}} = 1$. Thus with q_t sufficiently high, the unique equilibrium involves $s_{ct} = 1$ and $s_{dt} = 0$. More specifically,

$$q_t \geq 1 - \frac{\eta_c}{\eta_d} (1 + \gamma\eta_d)^{\varphi+1} (1 + \tau_t)^\varepsilon \left(\frac{A_{dt-1}}{A_{ct-1}}\right)^\varphi$$

is sufficient for the unique equilibrium (even when $\varepsilon \geq (2 - \alpha) / (1 - \alpha)$) to involve $s_{ct} = 1$ and $s_{dt} = 0$. This establishes Proposition 7. ■

Appendix E: Proof of Proposition 8

Using (7), (25), (53), (54), and (55), we obtain the optimal production of each input at time t as:

$$Y_{ct} = \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \frac{(1 + \tau_t)^\varepsilon A_{ct} A_{dt}^{\alpha+\varphi}}{\left(A_{dt}^\varphi + (1 + \tau_t)^{1-\varepsilon} A_{ct}^\varphi \right)^{\frac{\alpha}{\varphi}} (A_{ct}^\varphi + (1 + \tau_t)^\varepsilon A_{dt}^\varphi)} \quad (57)$$

$$Y_{dt} = \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct}^{\alpha+\varphi} A_{dt}}{\left(A_{dt}^\varphi + (1 + \tau_t)^{1-\varepsilon} A_{ct}^\varphi \right)^{\frac{\alpha}{\varphi}} (A_{ct}^\varphi + (1 + \tau_t)^\varepsilon A_{dt}^\varphi)} \quad (58)$$

so that the production of dirty input is decreasing in τ_t . Moreover, clearly $Y_{dt} \rightarrow 0$ as $\tau_t \rightarrow \infty$.

Strong substitutability case: Assume $\varepsilon > (1 - \alpha)^{-1}$. The proof involves three steps:

1. We show that a policy where Y_{ct} remains bounded cannot be optimal when the discount rate is sufficiently low
2. We show that if Y_{ct} becomes unbounded while Y_{dt} is bounded, Y_{ct} must go to infinity over time
3. We prove that the optimal policy involves a switch towards clean innovation only in finite time

First Step: To avoid a disaster it is necessary that Y_{dt} does not increase without bound over time ($Y_{dt} \leq (1 + \delta) \bar{S} / \xi$)

Consider first a policy that does not involve a permanent switch from dirty to clean production, that is, a policy where Y_{ct} remains bounded so that Y_t must remain bounded as well and thus so does C_t . We use the superscript *ns* (*ns* for "no switch") to denote the economic variables under this policy.

Now, consider an alternative policy which features a switch towards clean innovation only. Then as $\varepsilon > 1/(1 - \alpha)$, in the long-run Y_{dt} decreases to 0 in finite time. Thus S_t reaches \bar{S} in finite time and therefore in the long-run Y_t and C_t grow like A_{ct} . We use superscript *a* to denote economic variables under this alternative policy.

So there exists some time T and some consumption level \bar{C} and some $\theta > 0$, such that for $t \geq T$, consumption under the alternative policy keeps growing and is above $\bar{C} + \theta$ while consumption under the no switch policy remains below \bar{C} , and also by that time the environmental quality under the alternative policy has reached \bar{S} .

Note then that for $t \geq T$

$$u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns}) \geq u(C_t^a, \bar{S}) - u(\bar{C}, \bar{S})$$

which is positive and increasing in t .

Now

$$\begin{aligned}
W^a - W^{ns} &= \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) + \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) \\
&\geq \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) + \frac{1}{(1+\rho)^T} \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^{t-T}} (u(C_t^a, \bar{S}) - u(\bar{C}, \bar{S})).
\end{aligned}$$

As ρ becomes arbitrarily small the first sum is bounded below. If on the other hand, $u(C_t^a, \bar{S}) - u(\bar{C}, \bar{S})$ is positive and increases without bound, then the second sum increases without bound when ρ decreases. So for ρ sufficiently low $W^a - W^{ns}$ is necessarily positive: welfare is higher under the alternative policy than under the no switch policy.

Now if on the contrary $u(C, \bar{S})$ is bounded in C ,

$$u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns}) \geq u(\bar{C} + \theta, \bar{S}) - u(\bar{C}, \bar{S})$$

and

$$\begin{aligned}
W^a - W^{ns} &= \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) + \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) \\
&\geq \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} (u(C_t^a, S_t^a) - u(C_t^{ns}, S_t^{ns})) + \frac{1}{(1+\rho)^T} \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^{t-T}} (u(\bar{C} + \theta, \bar{S}) - u(\bar{C}, \bar{S})).
\end{aligned}$$

When ρ goes down to 0 the first sum is bounded below while the second sum increases without bound. So for ρ sufficiently low $W^a - W^{ns}$ is positive: welfare is higher under the alternative policy than under the no switch policy.

In both cases a policy where Y_{ct}/Y_{dt} remains finite cannot be optimal. This establishes Step 1.

Second step: We know that under the optimal policy Y_{ct} cannot remain bounded over time whereas Y_{dt} must remain bounded. Let $M > (1+\delta)\bar{S}/\xi$, then on the optimal path there must exist some t_1 and t_2 such that $Y_{ct_1} > kM$ and $Y_{ct_2} > kM$ with $t_2 > t_1$ and k is a positive number. Let us assume that for some periods τ between t_1 and t_2 , $Y_{c\tau} < M$. Then, since Y_{dt} is bounded above by $(1+\delta)\bar{S}/\xi$, it must be the case that during these periods C_τ is smaller than $2^{\frac{\varepsilon}{\varepsilon-1}}M$, since when $Y_{c\tau} < M$ and $Y_{d\tau} < M$, we must have $C_\tau < Y_\tau < 2^{\frac{\varepsilon}{\varepsilon-1}}M$. Now consider an alternative policy that would mimic the previous policy except that whenever $Y_{c\tau} < M$ under the original policy, the alternative policy induces $Y_{c\tau} > (k-1)M$ while maintaining the production of dirty input arbitrarily small. This in turn can be done because when M is sufficiently large, then at time t_1 the economy was already mostly relying on the clean sector (recall that $Y = \left(Y_c^{\frac{\varepsilon-1}{\varepsilon}} + Y_d^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, so that if Y_c is arbitrarily large compared to Y_d , Y_c/Y becomes arbitrarily close to 1 when $\varepsilon > 1$). Thus at time t_1 it would be possible to achieve a final output of at least $(k-1)M$ while maintaining the production of dirty input arbitrarily small. As no knowledge is lost over time, what was possible at time t_1 remains possible thereafter, hence it is still possible to achieve a final output of $(k-1)M$ with a

negligible amount of dirty input production at time $\tau > t_1$. Now by choosing k sufficiently large we make sure that consumption after t_1 is strictly higher than $2^{\frac{\varepsilon}{\varepsilon-1}} M$. Thus, under this alternative policy, consumption is higher during the time periods where the two policies differ and the quality of the environment is never lower under the alternative policy. Therefore the alternative policy beats the original policy, which proves by contradiction that on the optimal path, it must be the case that Y_{ct} remains higher than M between t_1 and t_2 . Consequently, Y_{ct} must always lie above M after t_1 . Since this is true for M arbitrarily large, Y_{ct} must grow to infinity over time.

Third step: Recall the equilibrium expressions (57) and (58). Thus when Y_{ct} tends towards infinity while Y_{dt} remains bounded, $\frac{Y_{ct}}{Y_{dt}} = \left((1 + \tau_t) \left(\frac{A_{ct}}{A_{dt}} \right)^{1-\alpha} \right)^\varepsilon$ must tends towards ∞ . But then we get that in the long-run

$$Y_{ct} \sim \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} A_{ct}$$

and

$$Y_{dt} \sim \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} A_{ct}^{\alpha+\varphi} A_{dt}^{\varepsilon(1-\alpha)} (1 + \tau)^{-\varepsilon}.$$

Thus

$$\begin{aligned} Y_t &\sim \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} A_{ct} \\ C_t &\sim \kappa A_{ct}, \end{aligned}$$

where κ is a constant.

Thus if some innovation was to be diverted away from the clean sector, the first order effect would be to reduce consumption, while increasing Y_{dt} (as $\varepsilon > (1 - \alpha)^{-1}$), thereby lowering environmental quality for all subsequent periods. This cannot be optimal, hence we get that in finite time, all innovation must occur in the clean sector.

As a consequence production of dirty input goes down to 0 in finite time, therefore environmental quality converges to \bar{S} in finite time, and the optimal profit tax and input taxes are temporary.

Weak substitutability case: If $1 < \varepsilon < 1/(1 - \alpha)$, then the input tax must be permanent, otherwise production of dirty input becomes unbounded.

Complementarity case: If $\varepsilon < 1$, the growth rate of the economy is the minimum of the growth rates of the two inputs, but the long-run growth rate of the dirty input must be zero in order to avoid a disaster, so the optimum does not involve a positive long-run growth rate. Policy intervention must then be permanent (without policy intervention there will be positive long-run growth). In fact, in this case both the input tax and the profit tax must be maintained permanently. Indeed if the input tax were to reach zero after some time T , the ratio $\frac{\eta_c \mu_c A_{ct-1}}{\eta_d \mu_d A_{dt-1}}$ would fail to internalize any environmental externality, and would lead to a pattern of innovation similar to what happens in the laissez-faire case with innovation in both sectors. But this would lead to a growth rate of $\gamma \tilde{\eta}$ higher than 0, yielding a contradiction. This establishes that the input tax cannot reach zero in any finite time T . Similarly, to induce

the optimal allocation of scientists, the knowledge externality needs to be internalized via a profit tax/subsidy which also needs to be maintained permanently. This completes the proof of Proposition 8. \blacksquare

Appendix F: Equilibrium profit ratio with exhaustible resources

We first analyze how the static equilibrium changes when we introduce the limited resource constraint. Thus here we drop subscript t for notational simplicity. The description of clean sectors remains exactly as before. Profit maximization by producers of machines in the dirty sector now leads to the equilibrium price $p_{di} = \frac{\psi}{\alpha_1}$ (as α_1 is the share of machines in the production of dirty input). The equilibrium output level for machines is then given by:

$$x_{di} = \left(\frac{(\alpha_1)^2 p_d R^{\alpha_2} L_d^{1-\alpha}}{\psi} \right)^{\frac{1}{1-\alpha_1}} A_{di} \quad (59)$$

Profit maximization by the dirty input producer leads to the following demand equation for the resource:

$$p_d \alpha_2 R^{\alpha_2-1} L_d^{1-\alpha} \int_0^1 A_{di}^{1-\alpha_1} x_{di}^{\alpha_1} di = c(Q)$$

Plugging in the equilibrium output level of machines (59) yields:

$$R = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_d}{c(Q)} \right)^{\frac{1-\alpha_1}{1-\alpha}} p_d^{\frac{1}{1-\alpha}} L_d \quad (60)$$

which in turn, together with (34), leads to the following expression for the equilibrium production of dirty input:

$$Y_d = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_d}{c(Q)} \right)^{\frac{\alpha_2}{1-\alpha}} p_d^{\frac{\alpha}{1-\alpha}} L_d A_d. \quad (61)$$

The equilibrium profits from producing machine i in the dirty sector becomes:

$$\pi_{di} = (1 - \alpha_1) \alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \left(\frac{1}{\psi^{\alpha_1}} \right)^{\frac{1}{1-\alpha_1}} p_d^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_d^{\frac{1-\alpha}{1-\alpha_1}} A_{di}.$$

The production of the clean input and the profits of the producer of machine i in the clean sector are still given by (18), that is:

$$Y_c = \left(\frac{\alpha^2}{\psi} p_c \right)^{\frac{\alpha}{1-\alpha}} L_c A_c \quad (62)$$

and profits from producing machines ci are

$$\pi_{ci} = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\psi} \right)^{\frac{\alpha}{1-\alpha}} p_c^{\frac{1}{1-\alpha}} L_c A_{ci}.$$

Next, labor market clearing requires that the marginal product of labor be equalized across sectors; this, together with (61) and (62), leads to the equilibrium price ratio (36): thus a higher extraction cost will bid up the price of the dirty input. Profit maximization by final good producer still yields (10) which, combined with (36), (61) and (62) yields the equilibrium labor share (equation (37)). Hence, the higher the extraction cost, the higher the amount of labor allocated to the clean industry when $\varepsilon > 1$, but the opposite holds when $\varepsilon < 1$.

The ratio of expected profits from undertaking innovation at time t in the clean versus the dirty sector, is then equal to (we reintroduce the time subscript):

$$\begin{aligned} \frac{\Pi_{ct}}{\Pi_{dt}} &= \frac{\eta_c}{\eta_d} \frac{(1 - \alpha_1) \alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \left(\frac{1}{\psi^{\alpha_1}} \right)^{\frac{1}{1-\alpha_1}}}{(1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\psi} \right)^{\frac{\alpha}{1-\alpha}}} \frac{p_{ct}^{\frac{1}{1-\alpha}} L_{ct}}{p_{dt}^{\frac{1}{1-\alpha_1}} R_t^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}}} \frac{A_{ct-1}}{A_{dt-1}} \\ &= \kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)} (1 + \gamma \eta_c s_{ct})^{-\varphi-1} A_{ct-1}^{-\varphi}}{(1 + \gamma \eta_d s_{dt})^{-\varphi_1-1} A_{dt-1}^{-\varphi_1}} \end{aligned}$$

where we let $\kappa \equiv \frac{(1-\alpha)\alpha}{(1-\alpha_1)\alpha_1^{\frac{1+\alpha_2-\alpha_1}{1-\alpha_1}}} \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}} \right)^{(\varepsilon-1)}$. This establishes (38). ■

Appendix G: Proof of Proposition 11

First, we derive the equilibrium production of R and Y_d .

Using both, the fact that the final good is chosen as numeraire and the expression for the equilibrium price ratio (36), we get:

$$\begin{aligned} p_c &= \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_d^{1-\alpha_1}}{\left((\alpha^{2\alpha} c(Q)^{\alpha_2})^{1-\varepsilon} A_c^\varphi + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{1-\varepsilon} A_d^{\varphi_1} \right)^{\frac{1}{1-\varepsilon}}} \\ p_d &= \frac{\alpha^{2\alpha} (c(Q))^{\alpha_2} A_c^{1-\alpha}}{\left((\alpha^{2\alpha} c(Q)^{\alpha_2})^{1-\varepsilon} A_c^\varphi + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{1-\varepsilon} A_d^{\varphi_1} \right)^{\frac{1}{1-\varepsilon}}} \end{aligned}$$

Similarly, using the expression for the equilibrium labor ratio (37), and labor market clearing (7), we obtain:

$$L_d = \frac{(c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^\varphi}{(c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^\varphi + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1}}$$

$$L_c = \frac{\left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{(1-\varepsilon)} A_d^{\varphi_1}}{(c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^\varphi + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{(1-\varepsilon)} A_d^{\varphi_1}}$$

Next, using the above expressions for equilibrium prices and labor allocation, and plugging them in (61) and (60), we obtain:

$$Y_d = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{\alpha_2}{1-\alpha}} \alpha^{2\alpha(\frac{1}{1-\alpha}-\varepsilon)} c(Q)^{-\varepsilon\alpha_2} A_c^{\alpha+\varphi} A_d^{\frac{1-\alpha_1}{1-\alpha}}}{\left((c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^\varphi + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{(1-\varepsilon)} A_d^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}$$

and

$$R = \frac{\alpha^{2\alpha(\frac{1}{1-\alpha}+1-\varepsilon)} \alpha_1^{2\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{1-\alpha_1}{1-\alpha}} \psi^{-\frac{\alpha_1}{1-\alpha}} (c(Q))^{\alpha_2-1-\alpha_2\varepsilon} A_c^{1+\varphi} A_d^{\frac{1-\alpha_1}{1-\alpha}}}{\left((c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^\varphi + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{(1-\varepsilon)} A_d^{\varphi_1}\right)^{\frac{1+\varphi}{\varphi}}},$$

so that:

$$\frac{R}{Y_d} = \frac{\alpha_2 \alpha^{2\alpha} (c(Q))^{\alpha_2-1}}{\left((\alpha^{2\alpha} c(Q)^{\alpha_2})^{1-\varepsilon} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{1-\varepsilon} \frac{A_d^{\varphi_1}}{A_c^\varphi}\right)^{\frac{1}{1-\varepsilon}}}.$$

In the remaining part of the proof, we again separately consider the case in which the two inputs are complements and the case where they are substitutes.

Substitutability case: When $\varepsilon > 1$, production of the dirty input is not essential to final good production. Thus, even if the stock of exhaustible resource gets fully depleted, it is still possible to achieve positive long-run growth. For a disaster to occur for any initial value of the environmental quality, it is necessary that Y_d grow at a positive rate while R must converge to 0. This implies that R/Y_d must converge to 0. This in turn means that the expression

$$\left(\alpha^{2\alpha} c(Q)^{\alpha_2}\right)^{1-\varepsilon} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{1-\varepsilon} \frac{A_d^{\varphi_1}}{A_c^\varphi}$$

must be equal to zero, which is impossible since $c(Q)$ is bounded above. Therefore, for sufficiently high initial quality of the environment, a disaster will be avoided.

Next, one can show that innovation will always end up occurring in the clean sector only. This is obvious if the resource gets depleted in finite time, so let us consider the case where it never gets depleted. The ratio of expected profits in clean versus dirty innovation is given by

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)} (1 + \gamma \eta_c s_{ct})^{-\varphi-1} A_{ct-1}^{-\varphi}}{(1 + \gamma \eta_d s_{dt})^{-\varphi_1-1} A_{dt-1}^{-\varphi_1}},$$

so that to prevent innovation from occurring asymptotically in the clean sector only it must be the case that $A_c^{-\varphi}$ does not grow faster than $A_{dt}^{-\varphi_1}$. In this case $R = O\left(A_d^{\frac{1-\alpha_1}{1-\alpha}}\right)$. But $A_d^{\frac{1-\alpha_1}{1-\alpha}}$ grows at a positive rate over time, so that the resource gets depleted in finite time after all.

Complementarity case: When $\varepsilon < 1$, Y_d is now essential for production and thus so is the resource flow R . Consequently, it is necessary that Q does not get depleted in finite time in order to get positive long-run growth. Recall that innovation takes place in both sectors if and only if $\kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)}(1+\gamma\eta_c s_{ct})^{-\varphi-1} A_{ct-1}^{-\varphi}}{(1+\gamma\eta_d s_{dt})^{-\varphi-1} A_{dt-1}^{-\varphi}} = 1$, and positive long-run growth requires positive growth of both dirty input and clean input productions. This requires that innovation occurs in both sectors, so $A_d^{(1-\alpha_1)}$ and $A_c^{(1-\alpha)}$ should be of same order.

But then:

$$R = O\left(A_d^{\frac{1-\alpha_1}{1-\alpha}}\right),$$

so that R grows over time. But this in turn leads to the resource stock being fully exhausted in finite time, thereby also shutting down the production of dirty input, which here prevents positive long-run growth. This completes the proof of Proposition 11. ■

Appendix H: Proof of Propositions 15 and 16

Proof of Proposition 15: Assume that the South operates under laissez-faire, whereas the North is subject to some environmental policy (τ_t^N, q_t^N) where $\tau_t^N \geq 0$, and the policy implies that at all point in time innovation occurs in the clean sector only in the North. North and South can freely trade the clean and dirty inputs (equation (47) is satisfied - which allows us to drop the superscript k for the price p_c^k). The global economy must satisfy labor market clearing in both countries

$$L_c^k + L_d^k = L^k, \text{ for } k = N, S, \quad (63)$$

as well as trade balance:

$$p_c \left(Y_c^k - \widetilde{Y}_c^k \right) + p_d^S \left(Y_d^k - \widetilde{Y}_d^k \right) = 0, \quad (64)$$

(where \widetilde{Y}_j^k denotes the consumption of input j in country k) and global market clearing for both input markets:

$$Y_j^N + Y_j^S = \widetilde{Y}_j^N + \widetilde{Y}_j^S. \quad (65)$$

What we need to show now is: first, that a disaster remains unavoidable under free trade when $1 < \varepsilon < 1/(1-\alpha)$ no matter the initial environmental quality; second, that when $\varepsilon > 1/(1-\alpha)$ a disaster may not be avoided any more under free trade even with an arbitrarily large \overline{S} .

The proof proceeds in four steps:

1. We describe the set of possible equilibria when the South has a comparative advantage in the production of the dirty input
2. We show that when $\alpha + \varphi > 0$, avoiding a disaster requires that the South has a comparative advantage in dirty input production.
3. Using steps 1 and 2, we prove that when $\alpha + \varphi > 0$, it is not possible to avoid a disaster under free trade either.

4. We derive explicit conditions under which redirecting innovation towards the clean sector in the North only, prevents a disaster under autarky but no longer under free trade in one equilibrium.

STEP 1

Lemma 3 *Assuming that the South has a comparative advantage in the production of dirty input, then in equilibrium the global economy will feature:*

- If $\frac{L^N}{L^S} > \frac{(A_d^N)^\varphi (1+\tau^N)^{\varepsilon+\frac{\alpha}{1-\alpha}} A_d^S}{(A_c^N)^\varphi A_d^N}$, non complete specialization in the North and complete specialization in dirty input production in the South
- If $\frac{(A_d^S)^\varphi A_c^S}{(A_c^S)^\varphi A_c^N} \leq \frac{L^N}{L^S} \leq \frac{(A_d^N)^\varphi (1+\tau^N)^{\varepsilon+\frac{\alpha}{1-\alpha}} A_d^S}{(A_c^N)^\varphi A_d^N}$ complete specialization in clean input production in the North and in dirty input production in the South
- If $\frac{L^N}{L^S} < \left(\frac{A_d^S}{A_c^S}\right)^\varphi \frac{A_c^S}{A_c^N}$ complete specialization in clean input production in the North and non complete specialization in the South

Proof. We consider each of these three cases in turn and derive necessary conditions for them to arise in equilibrium:

Case 1: incomplete specialization in the North, complete specialization in the South

In this case all labor in the South is devoted to the production of dirty input. Equation (45) and equalization of the MPL across sectors in the North leads to:

$$\frac{p_c}{p_d^N} = \left(\frac{A_d^N}{A_c^N}\right)^{(1-\alpha)}$$

From this we can express the equilibrium price levels as:

$$\begin{aligned} p_c &= \frac{(A_d^N)^{(1-\alpha)}}{\left((A_d^N)^\varphi + (1+\tau^N)^{1-\varepsilon} (A_c^N)^\varphi\right)^{\frac{1}{1-\varepsilon}}} \\ p_d^S &= (1+\tau^N) p_d^N = \frac{(1+\tau^N) (A_c^N)^{(1-\alpha)}}{\left((A_d^N)^\varphi + (1+\tau^N)^{1-\varepsilon} (A_c^N)^\varphi\right)^{\frac{1}{1-\varepsilon}}} \end{aligned}$$

Final good producer maximization then implies:

$$\frac{\widetilde{Y_c^N}}{\widetilde{Y_d^N}} = \frac{\widetilde{Y_c^S}}{\widetilde{Y_d^S}} = \left(\frac{p_c}{p_d^S}\right)^{-\varepsilon} = (1+\tau^N)^\varepsilon \left(\frac{A_c^N}{A_d^N}\right)^{(1-\alpha)\varepsilon} \quad (66)$$

Finally, (65), (63) and (64) yield the equilibrium allocation of labor between the two sectors:³⁸

$$\begin{aligned} L_c^N &= \frac{(1 + \tau^N)^\varepsilon (A_d^N)^\varphi \left(L^N + \frac{(1 + \tau^N)^{\frac{1}{1-\alpha}} A_d^S}{A_d^N} L^S \right)}{(1 + \tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi} \\ L_d^N &= \frac{(A_c^N)^\varphi \left(L^N - \frac{(A_d^N)^\varphi (1 + \tau^N)^{\varepsilon + \frac{1}{1-\alpha}} A_d^S}{A_d^N} L^S \right)}{(1 + \tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi} \end{aligned}$$

Non complete specialization in the North then imposes that $L_d^N > 0$, which in turn is equivalent to:

$$\frac{L^N}{L^S} > \frac{(A_d^N)^\varphi (1 + \tau^N)^{\varepsilon + \frac{1}{1-\alpha}} A_d^S}{(A_c^N)^\varphi A_d^N}. \quad (67)$$

Case 2: complete specialization in both countries

Here, all labor in the South is allocated to the production of dirty input, and all labor in the North is allocated to the production of the clean input. This yields equation (49). Complete specialization in clean production in the North then requires $\frac{MPL_c^N}{MPL_d^N} \geq 1$, whereas complete specialization in dirty production in the South requires $\frac{MPL_c^S}{MPL_d^S} \leq 1$. Finally, profit maximization by the final good producer yields the equilibrium price ratio:

$$\frac{p_c}{p_d^S} = \left(\frac{\widetilde{Y_c^k}}{\widetilde{Y_d^k}} \right)^{-\frac{1}{\varepsilon}} \text{ for } k = N, S. \quad (68)$$

³⁸Equations (64) and (66) yield the following expressions for the equilibrium input levels:

$$\begin{aligned} \widetilde{Y_d^N} &= \frac{p_c Y_c^N + p_d^S Y_d^N}{p_c (1 + \tau^N)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} + p_d^S} \\ \widetilde{Y_c^N} &= \left(1 + \tau^N \right)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} \frac{p_c Y_c^N + p_d^S Y_d^N}{p_c (1 + \tau^N)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} + p_d^S} \\ \widetilde{Y_d^S} &= \frac{p_d^S Y_d^S}{p_c (1 + \tau^N)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} + p_d^S} \\ \widetilde{Y_c^S} &= \left(1 + \tau^N \right)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} \frac{p_d^S Y_d^S}{p_c (1 + \tau^N)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} + p_d^S} \end{aligned}$$

now using this expression together with (65) and (18) leads to

$$\left(1 + \tau^N \right)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} (p_d^S)^{\frac{1}{1-\alpha}} A_d^S L^S + \left(1 + \tau^N \right)^\varepsilon \left(\frac{A_c^N}{A_d^N} \right)^{(1-\alpha)\varepsilon} (p_d^N)^{\frac{1}{1-\alpha}} A_d^N L_d^N = p_c^{\frac{1}{1-\alpha}} A_c^N L_c^N$$

from which we can infer the equilibrium allocation of labor between the two sectors..

This, together with (64), yields the following expression for the consumption of input j in country k :

$$\widetilde{Y}_d^k = \frac{p_c Y_c^k + p_d Y_d^k}{p_c^{1-\varepsilon} p_d^\varepsilon + p_d},$$

and, together with (65), we obtain the equilibrium price levels:

$$\begin{aligned} p_c &= \frac{(A_d^S L^S)^{\frac{1}{\varepsilon+1-\alpha}}}{\left((A_d^S L^S)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} + (A_c^N L^N)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} \right)^{\frac{1}{1-\varepsilon}}} \\ p_d^S &= \frac{(A_c^N L^N)^{\frac{1}{\varepsilon+1-\alpha}}}{\left((A_d^S L^S)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} + (A_c^N L^N)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} \right)^{\frac{1}{1-\varepsilon}}} \end{aligned} \quad (69)$$

Substituting for these equilibrium input price in the condition $\frac{MPL_c^N}{MPL_d^N} \geq 1$, implies that:

$$\frac{L^N}{L^S} \leq \frac{(A_d^N)^\varphi (1 + \tau^N)^{\varepsilon+\frac{\alpha}{1-\alpha}} A_d^S}{(A_c^N)^\varphi A_d^N} \quad (70)$$

and similarly substituting for the equilibrium input price in the condition $\frac{MPL_c^S}{MPL_d^S} \leq 1$, implies that:

$$\frac{L^N}{L^S} \geq \frac{(A_d^S)^\varphi A_c^S}{(A_c^S)^\varphi A_c^N}. \quad (71)$$

Using (49) and (69), we can derive the equilibrium input production in the South, namely:

$$Y_d^S = \left(\frac{(A_c^N L^N)^{\frac{1}{\varepsilon+1-\alpha}}}{\left((A_d^S L^S)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} + (A_c^N L^N)^{\frac{1-\varepsilon}{\varepsilon+1-\alpha}} \right)^{\frac{1}{1-\varepsilon}}} \right)^{\frac{\alpha}{1-\alpha}} A_d^S L^S. \quad (72)$$

Case 3: complete specialization in the North, non complete specialization in the South

The analysis is completely symmetric to that in case 1. We can derive the following expression for the equilibrium allocation of labor between the two sectors in the South:

$$L_d^S = \frac{(A_c^S)^\varphi \left(L^S + \frac{A_c^N}{A_c^S} L^N \right)}{(A_c^S)^\varphi + (A_d^S)^\varphi} \quad (73)$$

$$L_c^S = \frac{(A_d^S)^\varphi \left(L^S - \left(\frac{A_c^S}{A_d^S} \right)^\varphi \frac{A_c^N}{A_c^S} L^N \right)}{(A_c^S)^\varphi + (A_d^S)^\varphi} \quad (74)$$

so that this case scenario is possible only if

$$\frac{L^N}{L^S} \leq \left(\frac{A_d^S}{A_c^S} \right)^\varphi \frac{A_c^S}{A_c^N}.$$

We can then compute the equilibrium dirty input production level as

$$Y_d^S = \frac{(A_c^S)^{\alpha+\varphi} A_d^S \left(L^S + \frac{A_c^N}{A_c^S} L^N \right)}{((A_c^S)^\varphi + (A_d^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} \quad (75)$$

The argument so far establishes that as long as the South has a comparative advantage in the dirty input production, the above conditions do not overlap, hence they are not only necessary but also sufficient. This complete the proof of lemma. ■

STEP 2

Lemma 4 *When $\alpha + \varphi > 0$ any policy in the North aimed at avoiding a disaster would involve the South having a comparative advantage in dirty input production in the long-run.*

Proof. Suppose that the North had a comparative advantage in dirty input production in long-run. Then it would produce more dirty input for given taxes and technology under free-trade than in autarky. Under autarky, the production of dirty input in the North would become asymptotically proportional to $(1 + \tau_t^N)^{-\varepsilon} (A_{ct}^N)^{\alpha+\varphi}$. However, by assumption, $(A_{ct}^N)^{\alpha+\varphi}$ grows over time, therefore in order to avoid a disaster the government must impose a tax τ_t^N on dirty input production which increases without bound over time. At the same time, the fact that the North has a comparative advantage in dirty input production implies that

$$(1 + \tau_t^N)^{\frac{1}{1-\alpha}} \frac{A_{ct}^N}{A_{dt}^N} < \frac{A_{ct}^S}{A_{dt}^S}.$$

However, the fact that $\frac{A_{ct}^N}{A_{dt}^N} > 1$ whereas $\frac{A_{dt}^N}{A_{dt}^S}$ remains bounded above, makes it is impossible for this inequality to keep being satisfied over time meanwhile the tax schedule τ_t^N increases.

Now, suppose that there exists an infinite sequence of periods where the South would have a comparative advantage in clean input production and there exists an infinite sequence of periods where it has a comparative advantage in dirty input production. First, note that in the long-run imitation must asymptotically occur in the clean sector only. Indeed if imitation in the dirty sector kept on occurring indefinitely over time, then A_{dt}^S would tends towards A_{dt}^N so it would be impossible to satisfy $(1 + \tau_t^N)^{\frac{1}{1-\alpha}} \frac{A_{ct}^N}{A_{dt}^N} < \frac{A_{ct}^S}{A_{dt}^S}$ even when $\tau_t^N = 0$. Consequently, in the long-run A_{ct}^S should grow at the same rate as A_{ct}^N . But in periods where the South has a comparative advantage in dirty input production, its production will become arbitrarily large over time (this results from equations (75) and (72) and the fact that the condition to have complete specialization in the South but not in the North cannot hold in the long-run). Eventually this will lead to a disaster, since total production of dirty input must remain bounded to avoid a disaster.

Thus it is impossible to avoid a disaster through a policy where the South would not have a comparative advantage in dirty input production. ■

STEP 3

Lemma 5 *The North cannot prevent a disaster under free trade for any initial environmental quality when $\alpha + \varphi > 0$.*

Proof. From Lemma 3, we know that the North must have a comparative advantage in clean input production when $\alpha + \varphi > 0$ in order to avoid a disaster. Now we use Lemma 4, to describe the possible long-run scenarios and to show that in each of them the dirty input will grow positively.

A first possibility is to end up with complete specialization in the South but production of both inputs in the North. Indeed as long as the dirty input is the only one produced in the South, all imitation there occurs in the dirty sector, so A_{ct}^S does not grow over time. However $\frac{(A_d^N)^\varphi}{(A_c^N)^\varphi} \frac{(1+\tau^N)^{\varepsilon+1-\alpha} A_d^S}{A_d^N} \geq \frac{(A_d^N)^\varphi}{(A_c^N)^\varphi} \frac{A_d^S}{A_d^N}$ which grows exponentially, so the inequality $\frac{L^N}{L^S} \geq \frac{(A_d^N)^\varphi}{(A_c^N)^\varphi} \frac{(1+\tau^N)^{\varepsilon+1-\alpha} A_d^S}{A_d^N}$ must be violated at some point. Hence in finite time the economy will display either complete specialization in both countries case or complete specialization in the North only.

A second possibility is to end up with complete specialization in both countries. Then, asymptotically, using equation (72) we find that

$$Y_d^S \simeq (A_c^N L^N)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} (A_d^S L^S)^{\frac{\varepsilon(1-\alpha)}{\varepsilon(1-\alpha)+\alpha}},$$

so that Y_d^S grows exponentially at the strictly positive rate $(1 + \gamma\eta_c)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} - 1$. So a disaster cannot be avoided

A third possibility is to end up with no specialization in the South. Using equation (75), note that

$$Y_d^S > \frac{(A_c^S)^{\alpha+\varphi-1} A_d^S A_c^N L^N}{((A_c^S)^\varphi + (A_d^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} = O\left((A_c^S)^{\alpha+\varphi-1} A_c^N\right)$$

Now, given that $A_c^S < A_c^N$, then Y_d^S must at least be of the same order as $(A_c^N)^{\alpha+\varphi}$ which grows positively. Thus again, a disaster cannot be avoided.

Finally, if the economy moves back and forth between these latter two cases, and given that in either case the production of dirty input becomes unbounded, the policy cannot be successful either. This completes the proof. ■

STEP 4

Lemma 6 *If $\frac{A_{ct-1}^S}{(1-\kappa_d)A_{dt-1}^S + \kappa_d A_{dt-1}^N} < \min\left(\frac{(1+\gamma\eta_c)A_{ct-1}^N}{A_{dt-1}^N}, \left(\frac{(1+\gamma\eta_c)A_{ct-1}^N L^N}{A_{ct-1}^S L^S}\right)^{\frac{-1}{\varphi}}\right)$, then at time t there exists an equilibrium where the South has a comparative advantage in dirty technology and produces only in dirty technology. Moreover, $\frac{A_{ct}^S}{(1-\kappa_d)A_{dt}^S + \kappa_d A_{dt}^N} < \min\left(\frac{(1+\gamma\eta_c)A_{ct}^N}{A_{dt}^N}, \left(\frac{(1+\gamma\eta_c)A_{ct}^N L^N}{A_{ct}^S L^S}\right)^{\frac{-1}{\varphi}}\right)$*

Proof. Assume that at time t the South produces the dirty input only. Then all imitation in the South will be in the dirty input, hence

$$A_{dt}^S = (1 - \kappa_d) A_{dt-1}^S + \kappa_d A_{dt-1}^N.$$

This in turn implies that

$$\frac{A_{ct}^S}{A_{dt}^S} < \min\left(\frac{A_{ct}^N}{A_{dt}^N}, \left(\frac{A_{ct}^N L^N}{A_{ct}^S L^S}\right)^{\frac{-1}{\varphi}}\right),$$

so that it is indeed an equilibrium to have the South produce the dirty input only. Moreover, we have

$$\frac{A_{ct}^S}{(1 - \kappa_d) A_{dt}^S + \kappa_d A_{dt}^N} < \min \left(\frac{(1 + \gamma \eta_c) A_{ct}^N}{A_{dt}^N}, \left(\frac{(1 + \gamma \eta_c) A_{ct}^N L^N}{A_c^S L^S} \right)^{\frac{-1}{\varphi}} \right).$$

■

Now assume that

$$\frac{A_{c0}^S}{(1 - \kappa_d) A_{d0}^S + \kappa_d A_{d0}^N} < \min \left(\frac{(1 + \gamma \eta_c) A_{ct0}^N}{A_{dt0}^N}, \left(\frac{(1 + \gamma \eta_c) A_{ct0}^N L^N}{A_{c0}^S L^S} \right)^{\frac{-1}{\varphi}} \right),$$

which is satisfied for A_{c0}^S sufficiently small. Then there will be an equilibrium where the South always has a comparative advantage in the production of the dirty input and where the South completely specializes in dirty input production. At some point $\frac{(A_d^N)^\varphi}{(A_c^N)^\varphi} \frac{A_d^S}{A_d^N}$ will become sufficiently large that complete specialization must occur in equilibrium in the South. But then dirty input production is asymptotically equal to $(A_c^N L^N)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} (A_d^S L^S)^{\frac{\varepsilon(1-\alpha)}{\varepsilon(1-\alpha)+\alpha}}$, so that Y_d^S will keep increasing at rate $(1 + \gamma \eta_c)^{\frac{\alpha}{\varepsilon(1-\alpha)+\alpha}} - 1$. Thus in this case no matter the initial environmental quality, production of dirty input will keep increasing at a exponential rate over time leading for sure to a disaster. ■

Proof of Proposition 16: We now consider an equilibrium where the North always directs all its research efforts towards the clean technologies and puts a tax $\tau_t^N \geq 0$ on the production of dirty input, such that its production of dirty input under autarky is negligible. We also suppose that $\varphi + 1 < 0$ or equivalently $\varepsilon > (2 - \alpha) / (1 - \alpha)$.

The proof proceeds in four steps:

1. We show that under the condition on initial level of productivities, $\frac{A_{ct}^N}{A_{dt}^N} > \frac{A_{ct}^S}{A_{dt}^S}$ at all points in time, so that the North has a comparative advantage in clean input production.
2. We show that if the North produced only a negligible amount of dirty input under autarky, it must completely specialize in clean input production under free trade, so that world production of dirty input must be higher under free trade than under autarky.
3. We move one step back and compare the incentive to imitate in the South in clean technologies versus dirty technologies with free trade and without free trade
4. We show that the global production of dirty input is at least as high when one takes into account how opening up to trade modifies the dynamic response in the South as when one does not, so we conclude that the level of environmental quality is lower under free trade than under autarky

STEP 1

We want to show that if $\frac{A_{c0}^N}{A_{c0}^S} > \frac{1+\frac{\gamma\eta_c}{\kappa_c}}{1+\gamma\eta_c} > \frac{A_{d0}^N}{A_{d0}^S}$, then at all points in time we have: $\frac{A_{ct}^N}{A_{ct}^S} > \frac{A_{dt}^N}{A_{dt}^S}$. Given that the North devotes all its innovation efforts towards clean technologies, A_{dt}^N remains constant, and therefore $\frac{A_{dt}^N}{A_{dt}^S}$ is non increasing over time. Now, note that A_{ct}^S is smaller or equal to what it would be if imitation had always occurred in clean technologies in the South. If imitation indeed occurs in the clean sector only in the South, the law of motion for A_{ct}^S is given by

$$A_{ct}^S = (1 - \kappa_c) A_{ct-1}^S + \kappa_c A_{ct}^N,$$

so that

$$\frac{A_{ct}^S}{A_{ct}^N} = \frac{1 - \kappa_c}{1 + \gamma\eta_c} \frac{A_{ct-1}^S}{A_{ct-1}^N} + \kappa_c.$$

Now if $\frac{A_{ct-1}^S}{A_{ct-1}^N} < \frac{1+\gamma\eta_c}{1+\frac{\gamma\eta_c}{\kappa_c}}$, then we also get

$$\frac{A_{ct}^S}{A_{ct}^N} < \frac{1 + \gamma\eta_c}{1 + \frac{\gamma\eta_c}{\kappa_c}}.$$

Hence, starting from $\frac{A_{c0}^N}{A_{c0}^S} > \frac{1+\frac{\gamma\eta_c}{\kappa_c}}{1+\gamma\eta_c} > \frac{A_{d0}^N}{A_{d0}^S}$, we must get

$$\frac{A_{ct}^N}{A_{ct}^S} > \frac{1 + \frac{\gamma\eta_c}{\kappa_c}}{1 + \gamma\eta_c} > \frac{A_{d0}^N}{A_{d0}^S} > \frac{A_{dt}^N}{A_{dt}^S},$$

which is what we wanted to prove. As a consequence, no matter the tax policy in the North, the North keeps its comparative advantage in clean input production.

STEP 2

Under autarky, the expression for dirty input production in the North is given by

$$Y_d^{N,aut} = \frac{(A_c^N)^{\alpha+\varphi} A_d^N L^N}{\left((A_d^N)^\varphi + (1 + \tau^N)^{1-\varepsilon} (A_c^N)^\varphi \right)^{\frac{\alpha}{\varphi}} \left((1 + \tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi \right)}$$

Now assume that the condition (67) is satisfied, so that according to lemma 3 the economy will feature complete specialization in the North but not in the South. Then, using the fact that A_d^N is constant, that A_c^N is increasing and that A_d^S is non decreasing over time, production of

dirty input under autarky will satisfy:

$$\begin{aligned}
Y_d^{N,aut} &= \frac{(A_c^N)^{\alpha+\varphi} A_d^N L^N}{\left((A_d^N)^\varphi + (1+\tau^N)^{1-\varepsilon} (A_c^N)^\varphi\right)^{\frac{\alpha}{\varphi}} \left((1+\tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi\right)} \\
&> \frac{(A_c^N)^\alpha (1+\tau^N)^{\varepsilon+\frac{\alpha}{1-\alpha}} A_d^S (A_d^N)^\varphi L^S}{\left((A_d^N)^\varphi + (1+\tau^N)^{1-\varepsilon} (A_c^N)^\varphi\right)^{\frac{\alpha}{\varphi}} \left((1+\tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi\right)} \\
&> \frac{A_d^S L^S}{\left(1 + (1+\tau)^{-\varepsilon} \left(\frac{A_c^N}{A_d^N}\right)^\varphi\right)} \left((A_d^N)^\varphi (A_c^N)^{-\varphi} (1+\tau^N)^{\varepsilon-1} + 1\right)^{-\frac{\alpha}{\varphi}} \\
&> \frac{A_d^S L^S}{\left(1 + \left(\frac{A_c^N}{A_d^N}\right)^\varphi\right)} \left((A_d^N)^\varphi (A_c^N)^{-\varphi} + 1\right)^{-\frac{\alpha}{\varphi}} \\
&> \frac{(A_{d0}^N)^\varphi}{(A_{c0}^N)^\varphi} \left((A_d^N)^\varphi (A_{c0}^N)^{-\varphi} + 1\right)^{-\frac{\alpha}{\varphi}} A_{d0}^S L^S.
\end{aligned}$$

Thus, for a quality of the environment which is sufficiently low that the North would not want to produce as much dirty input as in autarky, this inequality cannot hold: the North must completely specialize in clean input production. So, we just need to check whether for given technologies, the world production of dirty input is higher than under autarky in equilibria that feature complete specialization in both countries or non complete specialization in the South.

Case of complete specialization in both countries

Dirty input production is then given by equation (72), while under autarky it was given by equation (21)

$$Y_d^{S,aut} = \frac{(A_c^S)^{\alpha+\varphi} (A_d^S) L^S}{((A_c^S)^\varphi + (A_d^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}}.$$

Now, using the fact that under complete specialization in the South equation (71) must hold, the difference between dirty input production in the South under free trade and autarky, must satisfy:

$$Y_d^{S,ft} - Y_d^{S,aut} > \frac{(A_c^S)^\alpha (A_d^S)^{\varphi+1}}{((A_d^S)^\varphi + (A_c^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} L^S.$$

Thus, provided that under autarky the North would not have produced more than this difference, world production of dirty input must have increased when moving from autarky to free-trade.

Now, using the fact that $\varphi + 1 > 0$, we see that A_d^S is non decreasing over time and

bounded above by A_{d0}^N whereas A_{c0}^S is non decreasing over time, therefore:

$$\begin{aligned} & \frac{(A_c^S)^\alpha (A_d^S)^{\varphi+1}}{((A_d^S)^\varphi + (A_c^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} L^S \\ & > \frac{(A_{d0}^N)^{\varphi+1} (A_{c0}^S)^\alpha L^S}{((A_{d0}^S)^\varphi + (A_{c0}^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}}. \end{aligned}$$

Thus if the quality of the environment is sufficiently low that under autarky the North would not produce more dirty input than $\frac{(A_{d0}^N)^{\varphi+1} (A_{c0}^S)^\alpha L^S}{((A_{d0}^S)^\varphi + (A_{c0}^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}}$, necessarily world production of dirty input has increased when moving from autarky to free trade.

Case of non complete specialization in South

Production of dirty input under free trade is then given by equation (75), so that

$$Y_d^{S,ft} - Y_d^{S,aut} = \frac{A_c^N}{A_c^S} \frac{(A_c^S)^{\alpha+\varphi} (A_d^S) L^N}{((A_c^S)^\varphi + (A_d^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}};$$

then, using the fact that the tax reduces the amount produced in the North under autarky and that at all points in time $\frac{A_c^N}{A_d^N} > \frac{A_c^S}{A_d^S}$, we get:

$$\begin{aligned} & Y_d^{S,ft} - (Y_d^{S,aut} + Y_d^{N,aut}) \\ &= \left(\frac{A_c^N}{A_c^S} \frac{(A_c^S)^{\alpha+\varphi} (A_d^S)}{((A_c^S)^\varphi + (A_d^S)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} - \frac{(A_c^N)^{\alpha+\varphi} A_d^N}{((A_d^N)^\varphi + (1 + \tau^N)^{1-\varepsilon} (A_c^N)^\varphi)^{\frac{\alpha}{\varphi}} ((1 + \tau^N)^\varepsilon (A_d^N)^\varphi + (A_c^N)^\varphi)} \right) L^N \\ &> \left(\frac{A_c^N}{A_c^S} \frac{(A_d^S)}{\left(1 + \left(\frac{A_d^S}{A_c^S}\right)^\varphi\right)^{\frac{\alpha+\varphi}{\varphi}}} - \frac{(A_c^N)^{\alpha+\varphi} A_d^N}{((A_d^N)^\varphi + (A_c^N)^\varphi)^{\frac{\alpha+1}{\varphi}} ((A_d^N)^\varphi + (A_c^N)^\varphi)} \right) L^N \\ &> \left(\frac{A_c^N}{A_c^S} \frac{(A_c^N)^{\alpha+\varphi} (A_d^S)}{((A_c^N)^\varphi + (A_d^N)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} - \frac{(A_c^N)^{\alpha+\varphi} A_d^N}{((A_d^N)^\varphi + (A_c^N)^\varphi)^{\frac{\alpha+\varphi}{\varphi}}} \right) L^N \\ &> 0 \end{aligned}$$

STEP 3

Under complete specialization the South will always imitate dirty technologies (as there is no demand for clean technologies).

Now let us consider the case where there is not complete specialization in the South. The relative incentives to imitate in the dirty versus the clean sector are pinned down by the profit ratio

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c (p_{ct}^S)^{\frac{1}{1-\alpha}} L_{ct}^S A_{ct}^N}{\kappa_d (p_{dt}^S)^{\frac{1}{1-\alpha}} L_{dt}^S A_{dt}^N}.$$

Now, taking into account the fact that the labor share is now given by equations (74) and (73), we get

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c (A_{ct}^S)^{-\varphi-1} A_{ct}^N A_{ct}^S L^S - \left(\frac{A_{ct}^S}{A_{dt}^S}\right)^\varphi A_{ct}^N L^N}{\kappa_d (A_{dt}^S)^{-\varphi-1} A_{dt}^N \frac{A_{ct}^S L^S + A_{ct}^N L^N}{A_{ct}^S L^S + A_{ct}^N L^N}},$$

which is identical to equation (43) except for the correction term $\frac{A_{ct}^S L^S - \left(\frac{A_{ct}^S}{A_{dt}^S}\right)^\varphi A_{ct}^N L^N}{A_{ct}^S L^S + A_{ct}^N L^N}$ which tilts imitation more towards the dirty sector under free trade than under autarky for given technologies.

Now, for $\varphi + 1 < 0$, (that is, $\varepsilon > (2 - \alpha) / (1 - \alpha)$) the incentive to imitate in dirty technology at time t is increasing in A_{dt-1}^S and decreasing in A_{ct-1}^S . So, for given technologies the incentive to imitate in dirty technologies, is always higher under free trade than under autarky. At the same time, the incentive to imitate in dirty technologies is higher the higher the current productivity of dirty technology and the lower the current productivity of clean technology: hence at all points, the incentive to imitate in dirty technologies is higher under free trade than under autarky. Hence A_{dt}^S is higher and A_{ct}^S lower under autarky than under free trade. So as long as scientists follow the same rule (for instance that of always choosing an equilibrium with imitation in the clean sector whenever such an equilibrium exists), they will always end up imitating dirty technologies more under free trade than under autarky.

STEP 4

Under complete specialization in both countries, we use equation (72) to get:

$$Y_d^{S,ft} = \frac{(A_c^N L^N)^{\frac{\alpha}{1-\varphi}} L^S (A_d^S)^{\frac{\varepsilon(1-\alpha)}{1-\varphi}}}{\left((L^S)^{\frac{\varphi}{1-\varphi}} + \left(\frac{A_c^N}{A_d^S} L^N\right)^{\frac{\varphi}{1-\varphi}}\right)^{\frac{\alpha}{\varphi}}},$$

which is increasing in A_d^S . Thus, taking into account the dynamic response of imitation in the South, production of dirty input under free trade is even higher under complete specialization than before taking this response into account.

Under incomplete specialization, we use equation (75) to get

$$Y_d^{S,ft} = \frac{(A_d^S) \left(L^S + \frac{A_c^N}{A_c^S} L^N\right)}{\left(1 + \left(\frac{A_d^S}{A_c^S}\right)^\varphi\right)^{\frac{\alpha+\varphi}{\varphi}}}$$

which is increasing in A_d^S and decreasing in A_c^S , so that in this case as well dirty input production is higher once we take into account the dynamic imitation response of the South.

This establishes that the environmental quality is reduced when moving from autarky to free trade, completing the proof of the propositions. \blacksquare

Appendix I: Characterization of global optimal environmental policy with no trade case

We now characterize the optimal policy from the point of view of a global social planner interested in maximizing the sum of the utilities of households in both countries (both given by (1)). This social planner will choose a dynamic path of final good production Y_t^k , consumption C_t^k , intermediary input productions Y_{jt}^k , machines production x_{jit}^k , labor share allocation L_{jt}^k , scientists allocation s_{jt}^k and quality of machines A_{jit}^k for each country $k = N, S$ and environmental quality S_t to maximize the Social Welfare Function

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left(L^N u \left(\frac{C_t^N}{L^N}, S_t \right) + L^S u \left(\frac{C_t^S}{L^S}, S_t \right) \right)$$

under the same constraints as for the baseline model, except that equation (9) becomes (41), and the productivity growth in the South equation (42).

Thus the maximization problem is very similar to that analyzed in subsection 3.2. One difference is that the shadow value of an environmental unit, which is identical in the two countries, now includes the marginal benefit to the utility of households in both countries so that:

$$\omega_t = \frac{1}{(1+\rho)^t} \left(L^N \frac{\partial u^N}{\partial S} + L^S \frac{\partial u^S}{\partial S} \right) + (1+\delta) \omega_{t+1}.$$

The social planner will still introduce a wedge $\omega_{t+1}\xi/\lambda_t^k$ between the price of the dirty input and its marginal product in the production of the final good. This wedge has the same interpretation as in the one country case; and thus it will be the higher (in absolute value) in the country with the lowest value λ_t^k , that is, the country with the lowest marginal utility of consumption (the rich country). This wedge translates into an optimal tax on the dirty input in country k :

$$\tau_t^k = \frac{\omega_{t+1}\xi}{\lambda_{dt}^k} = \frac{\omega_{t+1}\xi}{\lambda_t^k p_{dt}^k}$$

This expression is identical to that in the one-country case, and one can similarly establish that the optimal input tax will be temporary if the clean and dirty inputs are sufficiently close substitutes. But in addition, we now obtain the result that the optimal input tax in the North is higher than in the absence of the South, as ω is increased by the existence of a South.

Now, the comparison between the optimal input tax in the North and the South, is governed by the implicit expression already given above and derived in Appendix D, namely:

Proposition 17 *The global optimal input tax schedule (τ_t^N, τ_t^S) satisfies:*

$$\left(\frac{\lambda_t^k}{\omega_{t+1}\xi} \tau_t^k \right)^{1-\varepsilon} = 1 + \left(\frac{(A_{dt}^k)^{(1-\alpha)}}{(1 + \tau_t^k) (A_{ct}^k)^{(1-\alpha)}} \right)^{1-\varepsilon}.$$

In particular an increase in the relative productivity in the dirty sector in country k (A_{dt}^k/A_{ct}^k), a decrease in the marginal value of consumption λ_t^k or an increase in the shadow value of environment ω_{t+1} , either of these increases the tax τ_t^k in country k . The second effect will push towards a higher tax in the North, whereas (as long as the dirty sector is more advanced relative to the clean sector in the South than in the North), the first effect will push for a higher tax in the South. Without further assumptions either of these two effects may dominate, and in particular if the South lags far behind with respect to productivity in the clean sector, the dirty input tax may end up being higher in the South.

Define μ_{jt}^k as the Lagrange multiplier at time t for the growth equation for sector j in country k . The first-order condition with respect to A_{jt}^N now gives:

$$\mu_{jt}^N = \lambda_t^N \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) (p_{jt}^N)^{\frac{1}{1-\alpha}} L_{jt}^N + (1 + \gamma \eta_j s_{jt+1}^N) \mu_{jt+1}^N + \kappa_j s_{jt}^S \mu_{jt}^S \quad (76)$$

In words: the shadow value of one more unit of clean productivity is equal to its marginal product at time t (corresponding to the first term), plus its shadow value at time $t+1$ times $(1 + \gamma \eta_j s_{jt+1}^N)$ - the rate of productivity growth in the North between t and $t+1$ - (corresponding to the second term), plus an additional term $\kappa_c s_{jt}^S$ times the value of one unit of clean productivity in the South. This term did not exist in the closed economy, because it represents the international knowledge spillover: each additional unit of productivity in sector j in country N creates $\kappa_c s_{jt}^S$ units of productivity in sector j in country S .³⁹

The optimal allocation of scientists in the South will be governed by the comparison between the social gains from imitation in clean versus dirty technologies, namely $\mu_{ct}^S \kappa_c A_{ct}^N$ versus $\mu_{dt}^S \kappa_d A_{dt}^N$, and in the North it will be governed by the comparison between $\mu_{ct}^N \eta_c A_{ct-1}^N$ and $\mu_{dt}^N \eta_d A_{dt-1}^N$.

This analysis, combined with the same reasoning as for Proposition 8, establishes the following result (proof omitted):

³⁹The shadow value μ_{jt}^S is itself determined by first order conditions with respect to A_{jt}^S , which is

$$\mu_{jt}^S = \lambda_t^S \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) (p_{jt}^S)^{\frac{1}{1-\alpha}} L_{jt}^S + (1 - \kappa_j s_{jt+1}^S) \mu_{jt+1}^S \quad (77)$$

The interpretation is basically the same as for μ_{jt}^N : the shadow value of a unit of clean productivity is equal to its marginal product at time t , plus $1 - \kappa_j s_{jt+1}^S$ times its shadow value at time $t+1$ ($\kappa_j s_{jt+1}^S$ machines will adopt the technology in the North at time $t+1$: thus, the decision to allocate scientists to imitation in clean technologies in the South, is more "short sighted", that is with a higher weight on current profits, than if the North did not exist and the South had to innovate without benefiting from knowledge spillovers from the North), here, there is no technological spillover from South to North, hence the absence of a third term on the RHS of this equation (unlike in the previous equation for μ_{jt}^N).

Proposition 18 *The social optimum can be implemented through a combination of profits and input taxes both in the North and in the South, and a subsidy to machine consumers (to remove the monopoly distortion). If $\varepsilon > 1/(1 - \alpha)$ and the discount rate is sufficiently low, the optimal environmental taxes are temporary.*

6 Appendix J: Perfect competition in the absence of innovation

Here we show how our results are slightly modified if, instead of having monopoly rights randomly attributed to “entrepreneurs” when innovation does not occur, machines are produced competitively. There are two types of machines. Those where innovation occurred at the beginning of the period are produced monopolistically with demand function $x_{ji} = x_{ji}^m = \left(\frac{\alpha^2 p_j}{\psi}\right)^{\frac{1}{1-\alpha}} L_j A_{ji}$. Those for which innovation failed are produced competitively. In this case, machines are priced at marginal cost ψ , which leads to a demand for competitively produced machines equal to $x_{ji} = x_{ji}^c = \left(\frac{\alpha p_j}{\psi}\right)^{\frac{1}{1-\alpha}} L_j A_{ji}$. The number of machines produced under monopoly, is simply given by $\eta_j s_j$ (the number of successful innovation).

Hence the equilibrium production of input j is given by

$$\begin{aligned} Y_j &= L_j^{1-\alpha} \int_0^1 A_{ji}^{1-\alpha} (\eta_j s_j x_{ji,m}^\alpha + (1 - \eta_j s_j) x_{ji,c}^\alpha) di \\ &= \left(\frac{\alpha p_j}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \left(\eta_j s_j \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) A_j L_j \\ &= \left(\frac{\alpha p_j}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \widetilde{A}_j L_j \end{aligned}$$

where s_j is the number of scientists employed in clean industries and $\widetilde{A}_j = \left(\eta_j s_j \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) A_j$ is the average corrected productivity level in sector j (taking into account that some machines are produced by monopolists and others are not).

The equilibrium price ratio is now equal to:

$$\frac{p_c}{p_d} = \left(\frac{\widetilde{A}_c}{\widetilde{A}_d}\right)^{-(1-\alpha)},$$

and the equilibrium labor ratio becomes:

$$\frac{L_c}{L_d} = \left(\frac{\widetilde{A}_c}{\widetilde{A}_d}\right)^{-\varphi}.$$

The ratio of expected profits from innovation in clean versus dirty sector now becomes

$$\begin{aligned} \frac{\Pi_{ct}}{\Pi_{dt}} &= \frac{\eta_c}{\eta_d} \left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}} \frac{L_{ct} A_{ct-1}}{L_{dt} A_{dt-1}} \\ &= \frac{\eta_c}{\eta_d} \left(\frac{\left(\eta_c s_{ct} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) (1 + \gamma \eta_c s_{ct})}{\left(\eta_d s_{dt} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) (1 + \gamma \eta_d s_{dt})}\right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi} \end{aligned}$$

This yields the modified lemma:

Lemma 7 *In the decentralized equilibrium, innovation at time t can occur in the clean sector only when $\eta_c A_{ct-1}^{-\varphi} > \eta_d \left((1 + \gamma \eta_c) \left(\left(\eta_c \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) \right)^{\varphi+1} A_{dt-1}^{-\varphi} \right)$, in the dirty sector only when $\eta_c \left((1 + \gamma \eta_d) \left(\left(\eta_d \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) \right)^{\varphi+1} A_{ct-1}^{-\varphi} \right) < \eta_d A_{dt-1}^{-\varphi}$ and can occur in both when $\eta_c \left(\left(\eta_d s_{dt} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_d s_{dt}) \right)^{\varphi+1} A_{ct-1}^{-\varphi} = \eta_d \left(\left(\eta_c s_{ct} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_c s_{ct}) \right)^{\varphi+1} A_{dt-1}^{-\varphi}$.*

This modified lemma can then be used to prove the analogs of Propositions 1, 2 and 3 in the text. The results with exhaustible resource can similarly be generalized to this case.